DESIGN OF HIGHWAY BRIDGE ABUTMENTS AND FOUNDATIONS

PART 1

Project Manager: Juan Alcantar, P.E.

Michigan Department of Transportation

Submitted By:

Upul Attanayake, Ph.D., P.E.
Presidential Innovation Professor
Western Michigan University
(269) 276 - 3217
upul.attanayake@wmich.edu

Yufeng Hu, Ph.D., P.E.
Master Faculty Specialist
Western Michigan University
(269) 276 - 3310
yufeng.hu@wmich.edu

Western Michigan University
Department of Civil & Construction Engineering
College of Engineering and Applied Sciences
Kalamazoo, MI 49008-5316

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Section 4 Abutment with Piles

Section 1 Design Criteria

$kip := 1000lb$ $ksi := \frac{1000lb}{in^2}$ $ksf := \frac{kip}{ft^2}$ $kcf := 1\frac{kip}{ft^3}$ $psi := 1\frac{lb}{in^2}$

Legend: *The following formats and color coding are used to identify input variables, references, and results & checks presented in this document.*

User Input **References** Design Checks

Description

This example illustrates the design of an abutment with shallow and deep (pile) foundations for an interstate freeway bridge. The design is implemented in accordance with the Michigan Department of Transportation (MDOT) policies published as of 09/30/2022. The requirements of the 9^{th} Edition of the AASHTO LRFD Bridge Design Specification; as modified and supplemented by the Bridge Design Manual (BDM), Bridge Design Guides (BDG), and 2020 Standard Specifications for Construction (SSFC); are followed. Certain material and design parameters are selected to be in compliance with MDOT practice reflected in the Bridge Design System (BDS), the MDOT legacy software.

This step provides the design criteria, the bridge information, material properties, soil types and properties, and loads from the superstructure analysis.

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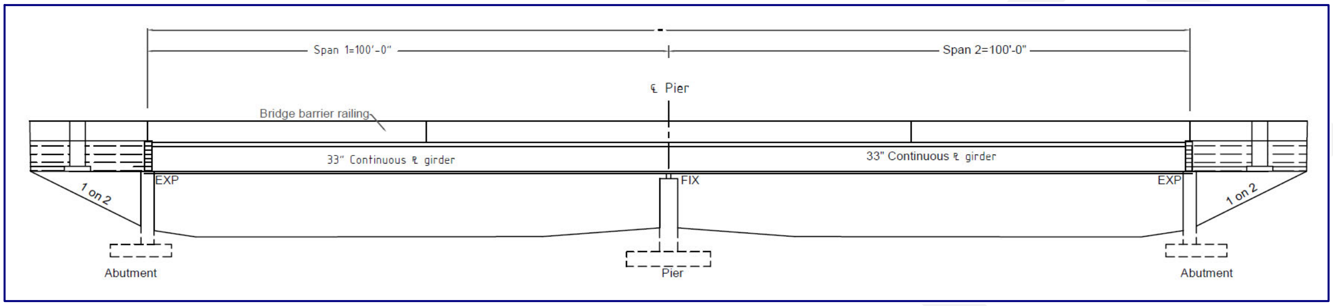
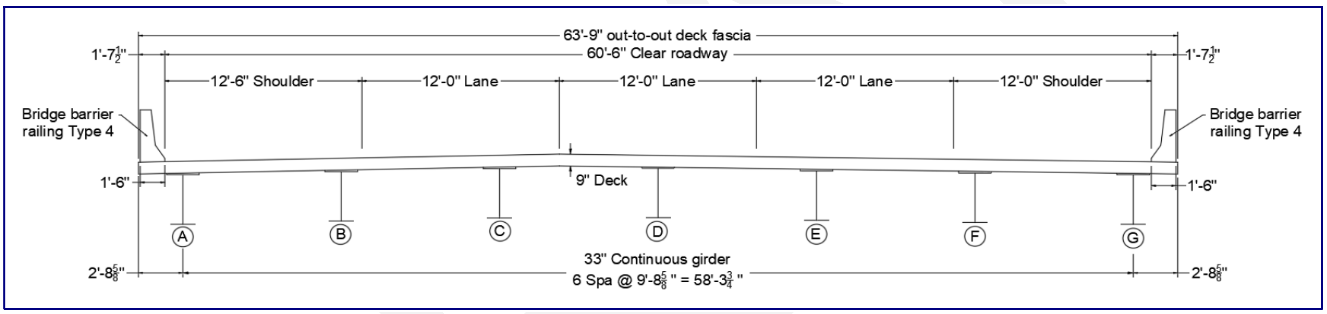
Bridge Information

This is a zero-skew, 200-ft long, two-span continuous, interstate freeway bridge. Each span consists of seven steel plate girders spaced at 9 ft - 8 $^5/_8$ in. on center. The vertical profile and typical cross-section of the bridge are shown below. The girders are designed for composite behavior with a 9-in. thick cast-in-place reinforced concrete deck to resist superimposed dead, live, and impact loads. The superstructure design is presented in the *Two-Span Continuous Bridge Steel Plate Girder Design Example* developed by Attanayake et al. (2021), which is cited in this example as the *Steel Plate Girder Design Example*.

Vertical Profile



Typical Cross-section



Bridge design span length	$L_{span} := 100 \cdot ft$	
Number of beams	$N_{beams} := 7$	
Beam spacing	$BeamSpacing := 9ft + 8.625in = 9.72\ ft$	
Out-to-out deck width	$W_{deck} := 63.75ft$	
Roadway clear width	$Rdwy_{width} := 60.5 \cdot ft$	
Number of design traffic lanes per roadway	$N_{lanes} := floor\left(\frac{Rdwy_{width}}{12 \cdot ft}\right) = 5$	**LRFD 3.6.1.1.1**
Deck slab thickness	$t_{Deck} := 9in$	**BDM 7.02.08**

Note: The type of barrier used in this example is for illustrative purposes only. It is the section used in the *Steel Plate Girder Design Example* to provide superstructure loads for this design. The BDG provides standard barrier section details.

Height of bridge railing	$h_{Railing} := 3ft + 4in = 3.33\,ft$	
Haunch thickness	$t_{Haunch} := 1in$	**BDM 7.02.19-C**
Overall depth of the girder at the abutment support	$d_{Girder} := 35in$	**Steel Plate Girder Design Example**

Material Properties

Reinforced concrete unit weight	$W_c := 150\dfrac{lb}{ft^3}$	
Concrete 28-day compressive strength	$f_c := 3ksi$	
Concrete density modification factor for normal weight concrete	$\lambda := 1$	**LRFD 5.4.2.8**
Concrete modulus of rupture	$f_r := 0.24\cdot\lambda\sqrt{\left(f_c\cdot ksi\right)} = 0.42\cdot ksi$	**LRFD 5.4.2.6**
Yield strength of reinforcing steel	$f_y := 60ksi$	
Concrete unit weight	$W_{con} := 145\dfrac{lb}{ft^3}$	**LRFD Table 3.5.1-1**
Correction factor for the source of aggregate	$K_1 := 1$	
Concrete modulus of elasticity	$E_c := 120000\cdot K_1\cdot\left(\dfrac{W_{con}}{1000\dfrac{lb}{ft^3}}\right)^2\cdot\left(\dfrac{f_c}{ksi}\right)^{0.33}\cdot ksi$ $E_c = 3.63\times10^3\cdot ksi$	**LRFD Eq. 5.4.2.4-1**
Steel modulus of elasticity	$E_s := 29000ksi$	

Nominal diameter and cross-section area of reinforcing steel bars

$$Dia(bar) := \begin{vmatrix} 0.5in & \text{if } bar = 4 \\ 0.625in & \text{if } bar = 5 \\ 0.75in & \text{if } bar = 6 \\ 0.875in & \text{if } bar = 7 \\ 1in & \text{if } bar = 8 \\ 1.128in & \text{if } bar = 9 \\ 1.27in & \text{if } bar = 10 \\ 1.41in & \text{if } bar = 11 \end{vmatrix} \qquad Area(bar) := \begin{vmatrix} 0.2in^2 & \text{if } bar = 4 \\ 0.31in^2 & \text{if } bar = 5 \\ 0.44in^2 & \text{if } bar = 6 \\ 0.6in^2 & \text{if } bar = 7 \\ 0.79in^2 & \text{if } bar = 8 \\ 1in^2 & \text{if } bar = 9 \\ 1.27in^2 & \text{if } bar = 10 \\ 1.56in^2 & \text{if } bar = 11 \end{vmatrix}$$

Reinforcing Steel Concrete Cover Requirements

BDG 5.16.01, 5.18.01, 5.22.01

The minimum concrete cover: 4 in. for the top and bottom of footing
3 in. for walls against soil

Backwall back cover $Cover_{bw} := 3in$

Abutment wall cover $Cover_{wall} := 3in$

Footing top and bottom cover $Cover_{ft} := 4in$

Soil Types and Properties

Bridge designers must interact closely with the Geotechnical Service Section since site conditions may make each substructure design unique.

Soil boring results showed the following soil profile. The Geotechnical Service Section uses this information to determine applicable bearing capacity, settlement, sliding resistance, etc.

Depth (ft)	Soil type	Total unit weight, γ_s (pcf)	ϕ', degree
0-25	Fine to coarse sands	120	30
25-75	Gravelly sands	125	36
75-90	Fine to coarse sands	120	30
90-130	Gravels	125	38

The groundwater table is not located within the vicinity of the foundation.

Unit weight of backfill soil $\gamma_s := 0.12kcf$

The active lateral earth pressure coefficient $k_a := 0.3$

Compacted Sand, LRFD Table 3.5.1-1

Loads from Superstructure

Dead Load

The superstructure dead load reactions at each girder end are taken from the *Steel Plate Girder Design Example*.

Dead load reactions at the exterior girder end supports

Table 12 of the Steel Plate Girder Design Example

Weight of structural components and non-structural attachments (DC) $R_{DCEx} := 44.6kip$

Weight of future wearing surface (DW) $R_{DWEx} := 8.0kip$

Dead load reactions at the interior girder end supports

Table 13 of the Steel Plate Girder Design Example

Weight of structural components and non-structural attachments (DC) $R_{DCIn} := 54.3kip$

Weight of future wearing surface (DW) $R_{DWIn} := 8.1kip$

Live Load

MDOT uses a modified version of the HL-93 loading specified in the LRFD Specifications. A single design truck load, a single 60-kip load (axle load), a two design truck load for continuous spans, and a design lane load are multiplied by a factor of 1.2 to make the design loading designated as HL-93 Mod.

Factor for HL-93 Mod	$f_{HL93Mod} := 1.2$	**BDM 7.01.04-A**
Dynamic load allowance	$IM := 0.33$	**LRFD Table 3.6.2.1-1**

According to the calculation presented in the *Steel Plate Girder Design Example,* the truck + lane load combination resulted in the maximum and minimum girder end reactions over the abutment. The unfactored girder support reactions for a single lane loaded case are listed below.

Maximum and minimum girder reactions due to truck load:

$V_{TruckMax} := 63.9kip$ $V_{TruckMin} := -5.9kip$ **Table A-4 of the Steel Plate Girder Design Example**

Maximum and minimum girder reactions due to lane load:

$V_{LaneMax} := 28.1kip$ $V_{LaneMin} := -3.5kip$ **Table A-4 of the Steel Plate Girder Design Example**

Section 2 Design of Abutment with a Spread Footing

Step 2.1 Preliminary Abutment Dimensions

Description

This step presents the selected preliminary abutment dimensions.

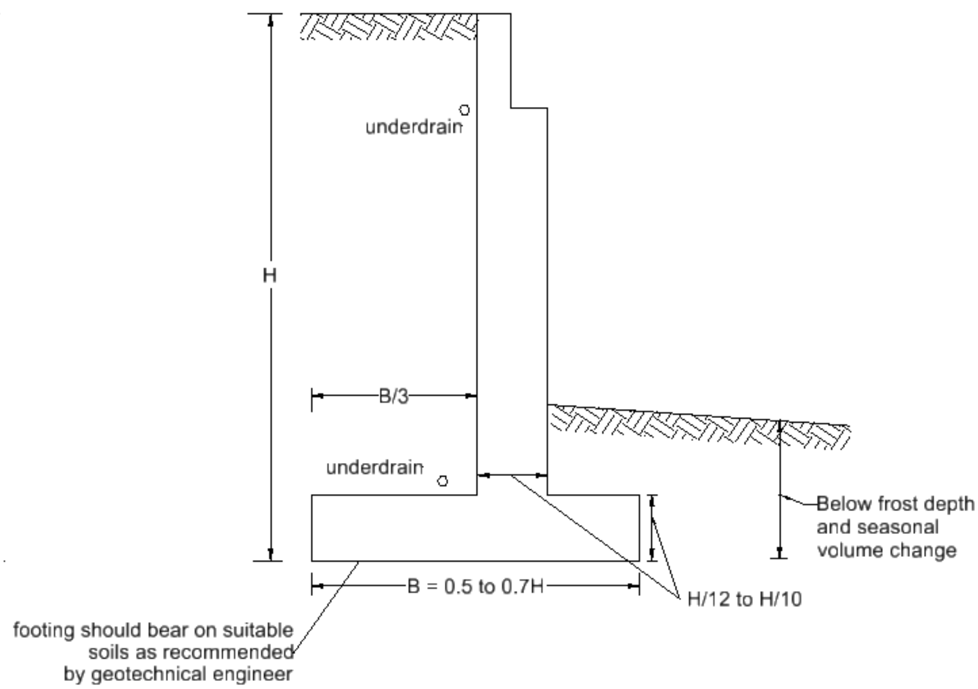
The selection of an optimal abutment type depends on the site conditions, cost considerations, superstructure geometry, and aesthetics. The common types include cantilever, counterfort, curtain wall, integral or semi-integral, and spill-through abutments. **BDM 7.03.01**

A concrete cantilever abutment is considered optimal for the selected site and the structure.

MDOT Bridge Design Manual lists the following minimum requirements:

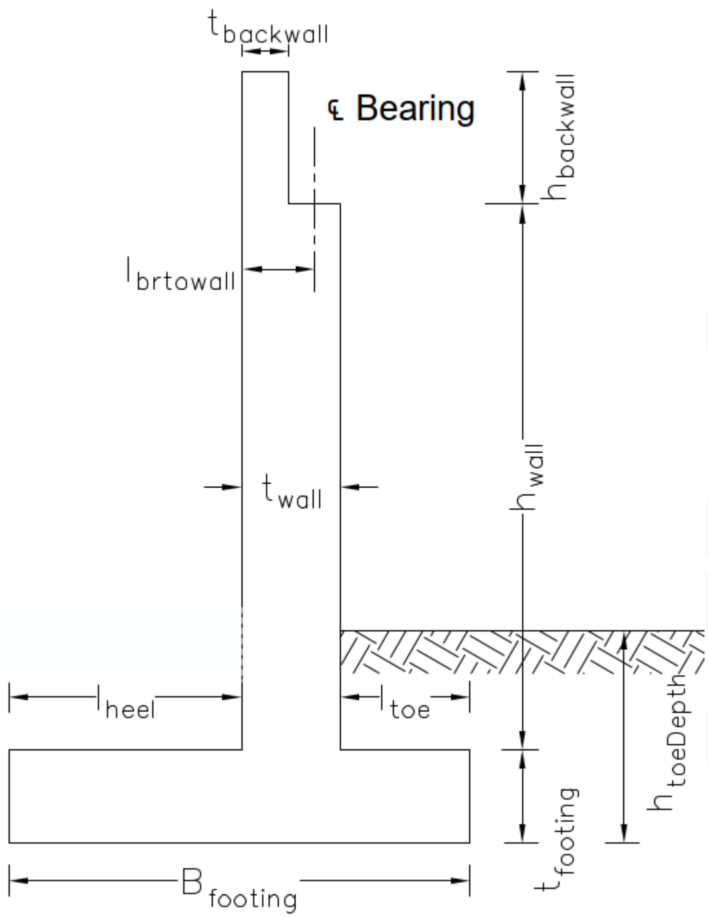
- The minimum wall thickness for abutments is 2 ft. **BDM 7.03.01C**
- The minimum thickness of footings is normally 2 ft - 6 in. When the wall thickness at its base becomes 3 ft or greater, the footing thickness is to be increased to 3 ft. Footing thickness is defined in 6 in. increments. **BDM 7.03.02A**
- The minimum footing width for cantilever abutments is 6 ft. **BDM 7.03.01B**

The designers select the preliminary dimensions based on state-specific standards and past experience. The preliminary footing dimensions are selected such that the resultant of the vertical loads falls within the middle one-third. As needed, the guidelines shown in the following figure can be used to establish the initial dimensions and check with the minimum requirements in the BDM.



Reference: Bowles, Foundation Analysis and Design. 5th Edition

The following figure shows the selected abutment geometry and dimensional variables:



The preliminary dimensions selected for this example are given below.

Abutment length $L_{abut} := W_{deck} = 63.75\ ft$

This abutment includes an independent cantilevered backwall, similar to the one shown in BDG 6.20.03A.

Backwall height $h_{backwall} := 4.25ft$

Backwall thickness $t_{backwall} := 1.5ft$

Abutment wall design height $h_{wall} := 17.54ft$

The thickness of an abutment wall is controlled by several factors including the space required to fit bearings and anchor bolts with an adequate edge distance. Since the bearing pad design is not included in this example, a 3ft-2in. thick abutment wall is selected by referring to a similar bridge to provide an adequate space to accommodate bearings and edge distances.

Abutment wall thickness $t_{wall} := 3ft + 2in = 3.17\ ft$

Distance from the toe to the front face of the abutment wall $l_{toe} := 4ft + 7in = 4.58\ ft$

Distance from the heel to the back face of the abutment wall	$l_{heel} := 9ft + 3in = 9.25\ ft$
Distance from center of the bearing pad to the back face of the abutment wall	$l_{brtowall} := 2ft + 4in = 2.33\ ft$
Footing width	$B_{footing} := l_{toe} + l_{heel} + t_{wall} = 17\ ft$
Footing length	$L_{footing} := 65.75ft$
Footing thickness	$t_{footing} := 3ft$
Toe fill depth to the bottom of the footing	$h_{toeDepth} := 7ft$

Note: Bottoms of footings are normally set 4 ft below the existing or proposed ground line to avoid frost heave. **BDM 7.03.02 D**

. Passive earth pressure is excluded from the footing design. **BDM 7.03.02 F**

Step 2.2 Application of Dead Load

Description

This step describes the application of the dead load on the abutment.

The common practice is to apply superstructure dead load as a uniformly distributed load over the length of the abutment. This is accomplished by adding exterior and interior girder end dead load reactions and dividing this quantity by the abutment length.

Dead load of superstructure

Weight of structural components and non-structural attachments (DC)

$$DC_{Sup} := \frac{2 \cdot R_{DCEx} + \left(N_{beams} - 2\right) \cdot R_{DCIn}}{L_{abut}} = 5.66 \cdot \frac{kip}{ft}$$

Weight of future wearing surface (DW)

$$DW_{Sup} := \frac{2 \cdot R_{DWEx} + \left(N_{beams} - 2\right) \cdot R_{DWIn}}{L_{abut}} = 0.89 \cdot \frac{kip}{ft}$$

Backwall weight

$$DC_{backwall} := h_{backwall} \cdot t_{backwall} \cdot W_c = 0.96 \cdot \frac{kip}{ft}$$

Abutment wall weight

$$DC_{wall} := h_{wall} \cdot t_{wall} \cdot W_c = 8.33 \cdot \frac{kip}{ft}$$

Footing weight

$$DC_{footing} := B_{footing} \cdot t_{footing} \cdot W_c = 7.65 \cdot \frac{kip}{ft}$$

Step 2.3 Application of Live Load

Description

This step describes the application of live load on the abutment.

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Live Load on the Backwall

The live load on the bridge has no impact on the backwall.
The live load on the approach slab is represented by a live load surcharge. The live load surcharge results in a lateral load on the backwall. Please refer to Step 2.4 for further details.

Live Load on the Abutment Wall

Depending on the number of design lanes, a multiple presence factor is applied to the HL-93 truck and lane loads.

$$MPF(lanes) := \begin{cases} 1.2 & \text{if } lanes = 1 \\ 1.0 & \text{if } lanes = 2 \\ 0.85 & \text{if } lanes = 3 \\ 0.65 & \text{otherwise} \end{cases}$$

LRFD Table 3.6.1.1.2-1

Live Load on Bridge Superstructure

The total live load girder end reactions is divided by the abutment length to calculate the load on a per foot basis.

Note: Even though the LRFD Specifications recommend including the dynamic impact in the design of substructures that are not completely buried, the MDOT practice is to exclude it from the design of bridge abutments.

$$lanes := 1 \qquad R_{LLWall1} := \frac{lanes \cdot (V_{TruckMax} + V_{LaneMax}) \cdot f_{HL93Mod} \cdot MPF(lanes)}{L_{abut}} = 2.08 \cdot \frac{kip}{ft}$$

$$lanes := 2 \qquad R_{LLWall2} := \frac{lanes \cdot (V_{TruckMax} + V_{LaneMax}) \cdot f_{HL93Mod} \cdot MPF(lanes)}{L_{abut}} = 3.46 \cdot \frac{kip}{ft}$$

$$lanes := 3 \qquad R_{LLWall3} := \frac{lanes \cdot (V_{TruckMax} + V_{LaneMax}) \cdot f_{HL93Mod} \cdot MPF(lanes)}{L_{abut}} = 4.42 \cdot \frac{kip}{ft}$$

$$lanes := 4 \qquad R_{LLWall4} := \frac{lanes \cdot (V_{TruckMax} + V_{LaneMax}) \cdot f_{HL93Mod} \cdot MPF(lanes)}{L_{abut}} = 4.5 \cdot \frac{kip}{ft}$$

$$lanes := 5 \qquad R_{LLWall5} := \frac{lanes \cdot (V_{TruckMax} + V_{LaneMax}) \cdot f_{HL93Mod} \cdot MPF(lanes)}{L_{abut}} = 5.63 \cdot \frac{kip}{ft}$$

The controlling live load on the abutment wall is

$$R_{LLWallMax} := \max(R_{LLWall1}, R_{LLWall2}, R_{LLWall3}, R_{LLWall4}, R_{LLWall5}) = 5.63 \cdot \frac{kip}{ft}$$

Live Load on Bridge Approach

The live load on the approach is represented by a surcharge load. This surcharge results in a lateral load on the abutment wall. Please refer to Step 2.4 for further details.

Live Load on the Footing

Live Load on Bridge Superstructure

The total live load girder reactions is divided by the footing length to calculate the load on a per foot basis. The dynamic impact is not included in the design of foundations. **LRFD 3.6.2.1**

$$lanes := 1 \qquad R_{LLFooting1} := \frac{lanes \cdot \left(V_{TruckMax} + V_{LaneMax}\right) \cdot f_{HL93Mod} \cdot MPF(lanes)}{L_{footing}} = 2.01 \cdot \frac{kip}{ft}$$

$$lanes := 2 \qquad R_{LLFooting2} := \frac{lanes \cdot \left(V_{TruckMax} + V_{LaneMax}\right) \cdot f_{HL93Mod} \cdot MPF(lanes)}{L_{footing}} = 3.36 \cdot \frac{kip}{ft}$$

$$lanes := 3 \qquad R_{LLFooting3} := \frac{lanes \cdot \left(V_{TruckMax} + V_{LaneMax}\right) \cdot f_{HL93Mod} \cdot MPF(lanes)}{L_{footing}} = 4.28 \cdot \frac{kip}{ft}$$

$$lanes := 4 \qquad R_{LLFooting4} := \frac{lanes \cdot \left(V_{TruckMax} + V_{LaneMax}\right) \cdot f_{HL93Mod} \cdot MPF(lanes)}{L_{footing}} = 4.37 \cdot \frac{kip}{ft}$$

$$lanes := 5 \qquad R_{LLFooting5} := \frac{lanes \cdot \left(V_{TruckMax} + V_{LaneMax}\right) \cdot f_{HL93Mod} \cdot MPF(lanes)}{L_{footing}} = 5.46 \cdot \frac{kip}{ft}$$

The controlling live load on the footing is

$$R_{LLFootingMax} := \max\left(R_{LLFooting1}, R_{LLFooting2}, R_{LLFooting3}, R_{LLFooting4}, R_{LLFooting5}\right) = 5.46 \cdot \frac{kip}{ft}$$

Live Load on Bridge Approach

Live load on the approach is accounted by a surcharge load. Please refer to Step 2.4 for further details.

Step 2.4 Application of Other Loads

Description

This step describes the application of braking force, wind load, earth load, and temperature load.

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Braking Force

Since the abutment in this example has expansion bearings, the horizontal component of the braking force is resisted by the fixed bearings located at the pier. Therefore, a horizontal braking force is not applied at the abutment. The braking force calculation is presented in Appendix 2.A and the pier design example.

Note: Although there is a possibility to develop a vertical force component at the bearings due to the braking force applied at 6 ft above the bridge deck, MDOT practice is to exclude this load from substructure design.

Wind Load

Since the abutment has expansion bearings, the longitudinal component of the wind load on superstructure is resisted by the fixed bearings at the pier. The relevant calculations are presented in the pier design example.

Note: Although the transverse component of the wind load acts on the abutment, it is often small and does not impact the design. The MDOT practice is to exclude this load from the abutment design. The wind load calculation is described in the pier design example.

Earth Load

The earth load includes lateral earth pressure, live load surcharge, and vertical earth pressure on the footing. As per the geotechnical engineer, the groundwater table is not located in the vicinity of the foundation. Therefore, the effect of hydrostatic pressure is excluded. Hydrostatic pressure should be avoided if possible in all abutment and retaining wall design cases through the design of an appropriate drainage system.

Lateral Load Due to Lateral Earth Pressure

The lateral pressure and the resultant load are calculated. This load acts at a distance of one third of the height from the base of the components being investigated.

Backwall

Lateral earth pressure at the base $p_{bw} := k_a \cdot \gamma_s \cdot h_{backwall} = 0.15 \cdot ksf$ **LRFD Eq. 3.11.5.1-1**

Lateral load $P_{EHBackwall} := \frac{1}{2} \cdot p_{bw} \cdot h_{backwall} = 0.33 \cdot \frac{kip}{ft}$

Abutment Wall

Lateral earth pressure at the base $p_{wall} := k_a \cdot \gamma_s \cdot \left(h_{backwall} + h_{wall}\right) = 0.78 \cdot ksf$

Lateral load $P_{EHWall} := \frac{1}{2} \cdot p_{wall} \cdot \left(h_{backwall} + h_{wall}\right) = 8.55 \cdot \frac{kip}{ft}$

Footing

Lateral earth pressure at the base $p_{ft} := k_a \cdot \gamma_s \cdot \left(h_{backwall} + h_{wall} + t_{footing}\right) = 0.89 \cdot ksf$

Lateral load $P_{EHFooting} := \frac{1}{2} \cdot p_{ft} \cdot \left(h_{backwall} + h_{wall} + t_{footing}\right) = 11.06 \cdot \frac{kip}{ft}$

Vertical Earth Load on the Footing

Back side (heel) $EV_{earthBk} := \gamma_s \cdot l_{heel} \cdot \left(h_{backwall} + h_{wall}\right) = 24.19 \cdot \frac{kip}{ft}$

Front side (toe) $EV_{earthFt} := \gamma_s \cdot l_{toe} \cdot \left(h_{toeDepth} - t_{footing}\right) = 2.2 \cdot \frac{kip}{ft}$

Live Load Surcharge

Live load surcharge is applied to account for a vehicular load acting on the backfill surface within a distance equal to one-half the wall height behind the back face of the wall. **LRFD 3.11.6.4**

Height of the abutment $h_{backwall} + h_{wall} + t_{footing} = 24.79\ ft$

Note: The equivalent height of soil for the surcharge load is defined as a function of the abutment height.

Equivalent height of soil for the surcharge load $h_{eq} := 2ft$ **LRFD Table 3.11.6.4-1**

Lateral surcharge pressure $\sigma_p := k_a \cdot \gamma_s \cdot h_{eq} = 0.07 \cdot ksf$ **LRFD Eq. 3.11.6.4-1**

Backwall

Lateral load $P_{LSBackwall} := \sigma_p \cdot h_{backwall} = 0.31 \cdot \frac{kip}{ft}$

Abutment Wall

Lateral load $P_{LSWall} := \sigma_p \cdot \left(h_{backwall} + h_{wall}\right) = 1.57 \cdot \frac{kip}{ft}$

Footing

Lateral load $P_{LSFooting} := \sigma_p \cdot \left(h_{backwall} + h_{wall} + t_{footing}\right) = 1.78 \cdot \frac{kip}{ft}$

Vertical load $V_{LSFooting} := \gamma_s \cdot l_{heel} \cdot h_{eq} = 2.22 \cdot \frac{kip}{ft}$

Temperature Load

The forces transferred from the superstructure to the substructure due to temperature is influenced by the shear stiffness of the bearing pads.

Thermal expansion coefficient of steel (/°F) $\alpha := 6.5 \cdot 10^{-6}$

Note: MDOT uses a 45° F drop and 35° F rise from the temperature at the time of construction. **BDM 7.01.07 cold climate temperature range**

Contraction and expansion temperatures $T_{contraction} := 45$ $T_{expansion} := 35$

Bridge superstructure contraction $\Delta_{TContr} := \alpha \cdot L_{span} \cdot T_{contraction} = 0.35 \cdot in$

Bridge superstructure expansion $\Delta_{TExp} := \alpha \cdot L_{span} \cdot T_{expansion} = 0.27 \cdot in$

Shear modulus of the elastomer $G_{bearing} := 100 \frac{lb}{in^2}$ **BDM 7.02.05C**

Plan view area of the bearing pad $A_{bearing} := 22in \cdot 9in = 198 \cdot in^2$

Total elastomer thickness $h_{rt} := 2.75in$

Since the pier bearings are fixed, the total superstructure deformation is imposed on the abutment bearings.

The force acting on a bearing due to superstructure contraction

$$H_{buContr} := \frac{G_{bearing} \cdot A_{bearing} \cdot \Delta_{TContr}}{h_{rt}} = 2.53 \cdot kip$$

LRFD Eq. 14.6.3.1-2

Total force acting on the abutment due to superstructure contraction

$$TU_{Contr} := \frac{N_{beams} \cdot H_{buContr}}{L_{abut}} = 0.28 \cdot \frac{kip}{ft}$$

The force acting on a bearing due to superstructure expansion

$$H_{buExp} := \frac{G_{bearing} \cdot A_{bearing} \cdot \Delta_{TExp}}{h_{rt}} = 1.97 \cdot kip$$

LRFD Eq. 14.6.3.1-2

Total force acting on the abutment due to superstructure expansion

$$TU_{Exp} := \frac{N_{beams} \cdot H_{buExp}}{L_{abut}} = 0.22 \cdot \frac{kip}{ft}$$

Step 2.5 Combined Load Effects

Description

This step presents the procedure for combining all load effects and calculating total factored forces and moments acting at the base of the backwall, abutment wall, and footing.

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Strength I, Strength III, Strength V, and Service I limit states are considered for the analysis and design of an abutment.

LRFD 3.4.1

Strength I = 1.25DC + 1.5DW + 1.75LL + 1.75BR + 1.5EH + 1.35EV + 1.75LS + 0.5TU

Strength III = 1.25DC + 1.5DW + 1.5EH + 1.35EV + 1.0WS + 0.5TU

Strength V = 1.25DC + 1.5DW + 1.35LL + 1.35BR + 1.0WS + 1.0WL + 1.5EH + 1.35EV + 1.35LS + 0.5TU

Service I = 1.0DC + 1.0DW + 1.0LL + 1.0BR + 1.0WS + 1.0WL + 1.0EH + 1.0EV + 1.0LS + 1.0TU

BR = vehicular braking force
DC = dead load of structural components and nonstructural attachments
DW = dead load of future wearing surface and utilities
EH = horizontal earth pressure load
EV = vertical pressure from the earth fill
LL = vehicular live load
LS = live load surcharge
WL = wind on live load
WS = wind load on structure
TU = force effect due to uniform temperature

Limit states that are not shown either do not control or are not applicable. Generally, Strength III or Strength V may control the design of abutments with fixed bearings when the wind load is considered.

Note: Since the MDOT practice is to exclude wind load from the abutments design, only Strength I and Service I limit states are included in this section.

Four load cases are considered in the design of an abutment:

BDM 7.03.01

Case I Construction state: abutment built and backfilled to grade.
Case II Bridge open to traffic with traffic loading on the approach only.
Case III Bridge with traffic on it and no load on the approach.
Case IV Contraction: Loading forces of Case II plus the effects of temperature contraction in the deck transmitted to the abutment.

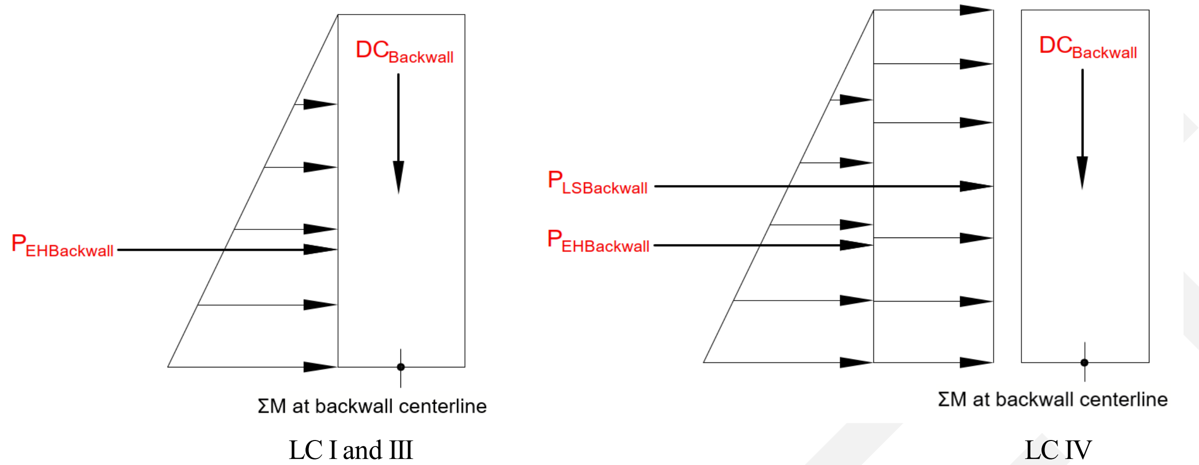
Since Case IV always governs over Case II for the bridge abutment selected for this example (independent cantilever abutment), only Cases I, III, and IV are considered.

The temperature force in Load Case IV is due to contraction, therefore

$$TU := TU_{Contr} = 0.28 \cdot \frac{kip}{ft}$$

The base of the backwall, the base of the abutment wall, and the base of the footing are the three critical locations where the force effects need to be combined and analyzed for the design of an abutment. Horizontal loads parallel to the longitudinal axis of the abutment are not considered for backwall and abutment wall design because of the high moment of inertia about the longitudinal axis of the bridge. However, such loads, even though relatively small, are considered at the base of the footing.

Forces and Moments at the Base of the Backwall



Strength I

Strength I = 1.25DC + 1.5DW + 1.75LL + 1.75BR + 1.5EH + 1.35EV + 1.75 LS + 0.5TU

Load Case I

Factored vertical force $F_{VBwLC1StrI} := 1.25 \cdot DC_{backwall} = 1.2 \cdot \frac{kip}{ft}$

Factored shear force parallel to the transverse axis of the backwall $V_{uBwLC1StrI} := 1.5 \cdot P_{EHBackwall} = 0.49 \cdot \frac{kip}{ft}$

Factored moment about the longitudinal axis of the backwall $M_{uBwLC1StrI} := 1.5 \cdot P_{EHBackwall} \cdot \frac{h_{backwall}}{3} = 0.69 \cdot \frac{kip \cdot ft}{ft}$

Load Case III

Factored vertical force $F_{VBwLC3StrI} := 1.25 \cdot DC_{backwall} = 1.2 \cdot \frac{kip}{ft}$

Factored shear force parallel to the transverse axis of the backwall $V_{uBwLC3StrI} := 1.5 \cdot P_{EHBackwall} = 0.49 \cdot \frac{kip}{ft}$

Factored moment about the longitudinal axis of the backwall $M_{uBwLC3StrI} := 1.5 \cdot P_{EHBackwall} \cdot \frac{h_{backwall}}{3} = 0.69 \cdot \frac{kip \cdot ft}{ft}$

Load Case IV

Factored vertical force $F_{VBwLC4StrI} := 1.25 \cdot DC_{backwall} = 1.2 \cdot \frac{kip}{ft}$

Factored shear force parallel to the transverse axis of the backwall $V_{uBwLC4StrI} := 1.5 \cdot P_{EHBackwall} + 1.75 \cdot P_{LSBackwall} = 1.02 \cdot \frac{kip}{ft}$

Factored moment about the longitudinal axis of the backwall

$$M_{uBwLC4StrI} := 1.5 \cdot P_{EHBackwall} \cdot \frac{h_{backwall}}{3} + 1.75 \cdot P_{LSBackwall} \cdot \frac{h_{backwall}}{2} = 1.83 \cdot \frac{kip \cdot ft}{ft}$$

Service I

Service I = 1.0DC + 1.0DW + 1.0LL + 1.0BR + 1.0WS + 1.0WL + 1.0EH + 1.0EV + 1.0LS + 1.0TU

Load Case IV related calculations are shown below since it controls the Service I limit.

Factored vertical force $F_{VBackwallSerI} := DC_{backwall} = 0.96 \cdot \frac{kip}{ft}$

Factored shear force parallel to the transverse axis of the backwall $V_{uBackwallSerI} := P_{EHBackwall} + P_{LSBackwall} = 0.63 \cdot \frac{kip}{ft}$

Factored moment about the longitudinal axis of the backwall

$$M_{uBackwallSerI} := P_{EHBackwall} \cdot \frac{h_{backwall}}{3} + P_{LSBackwall} \cdot \frac{h_{backwall}}{2}$$

$$M_{uBackwallSerI} = 1.11 \cdot \frac{kip \cdot ft}{ft}$$

Summary of Forces and Moments at the Base of the Backwall

Factored vertical force, F_{VBw} (kip/ft)

	Strength I	Service I
LC I	1.20	-
LC III	1.20	-
LC IV	1.20	0.96

Factored shear force parallel to the transverse axis of the backwall, V_{uBw} (kip/ft)

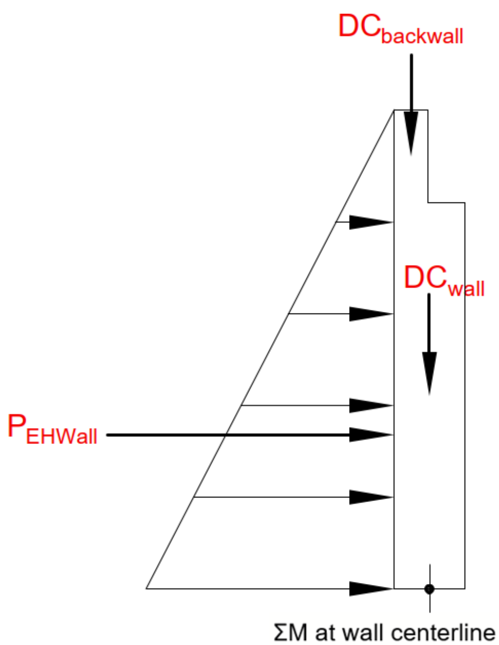
	Strength I	Service I
LC I	0.49	-
LC III	0.49	-
LC IV	1.02	0.63

Factored moment about the longitudinal axis of the backwall, M_{uBw} (kip·ft/ft)

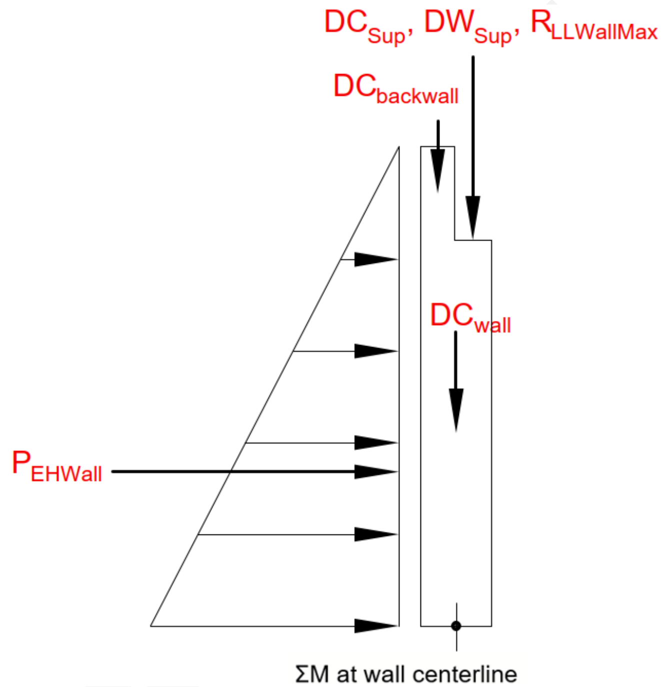
	Strength I	Service I
LC I	0.69	-
LC III	0.69	-
LC IV	1.83	1.11

Forces and Moments at the Base of the Abutment Wall

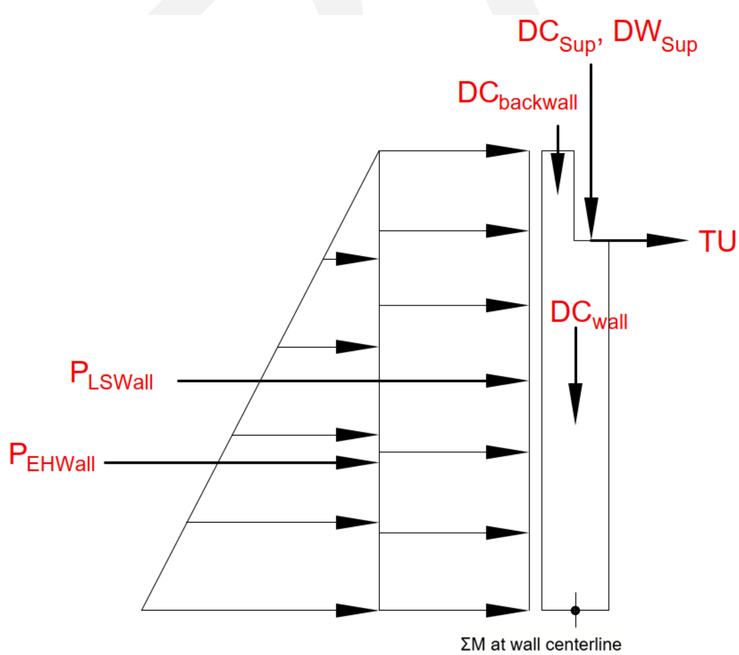
Load Cases I, III, and IV are considered. The superstructure dead and live loads and the uniform temperature induced loads are considered in addition to the backwall and abutment wall dead loads, lateral earth pressure, and lateral surcharge pressure.



LC I



LC III



LC IV

Strength I

Strength I = 1.25DC + 1.5DW + 1.75LL + 1.75BR + 1.5EH + 1.35EV + 1.75 LS + 0.5TU

Load Case I

Factored vertical force

$$F_{VWallLC1StrI} := 1.25 \cdot \left(DC_{backwall} + DC_{wall}\right) = 11.61 \cdot \frac{kip}{ft}$$

Factored shear force parallel to the transverse axis of the abutment wall

$$V_{uWallLC1StrI} := 1.5 \cdot P_{EHWall} = 12.82 \cdot \frac{kip}{ft}$$

The backwall weight reduces the critical moment at the base of the abutment wall. This requires using the minimum load factor of 0.9 for the dead load (DC) instead of the factor 1.25 in the Strength I combination.

LRFD 3.4.1
LFRD Table 3.4.1-2

Similar conditions are applied for the moments calculated about the longitudinal axis of the abutment wall for all the load cases and all the limit states.

Factored moment about the longitudinal axis of the abutment wall

$$M_{uWallLC1StrI} := 0.9 \cdot DC_{backwall} \cdot \frac{\left(t_{backwall} - t_{wall}\right)}{2} + 1.5 \cdot P_{EHWall} \cdot \frac{\left(h_{backwall} + h_{wall}\right)}{3} = 92.4 \cdot \frac{kip \cdot ft}{ft}$$

Load Case III

Factored vertical force

$$F_{VWallLC3StrI} := 1.25 \cdot \left(DC_{Sup} + DC_{backwall} + DC_{wall}\right) + 1.5DW_{Sup} + 1.75R_{LLWallMax}$$

$$F_{VWallLC3StrI} = 29.86 \cdot \frac{kip}{ft}$$

Factored shear force parallel to the transverse axis of the abutment wall

$$V_{uWallLC3StrI} := 1.5 \cdot P_{EHWall} = 12.82 \cdot \frac{kip}{ft}$$

Factored moment about the longitudinal axis of the abutment wall

$$M_{uWallLC3StrI} := 0.9 \cdot DC_{backwall} \cdot \frac{\left(t_{backwall} - t_{wall}\right)}{2} \dots$$
$$+ \left(1.25 \cdot DC_{Sup} + 1.5 \cdot DW_{Sup} + 1.75 \cdot R_{LLWallMax}\right) \cdot \left(l_{brtowall} - \frac{t_{wall}}{2}\right) \dots$$
$$+ 1.5 \cdot P_{EHWall} \cdot \frac{\left(h_{backwall} + h_{wall}\right)}{3}$$

$$M_{uWallLC3StrI} = 106.09 \cdot \frac{kip \cdot ft}{ft}$$

Load Case IV

Factored vertical force

$$F_{VWallLC4StrI} := 1.25 \cdot \left(DC_{Sup} + DC_{backwall} + DC_{wall}\right) + 1.5DW_{Sup} = 20.01 \cdot \frac{kip}{ft}$$

Factored shear force parallel to the transverse axis of the abutment wall

$$V_{uWallLC4StrI} := 1.5 \cdot P_{EHWall} + 1.75 \cdot P_{LSWall} + 0.5TU = 15.7 \cdot \frac{kip}{ft}$$

Factored moment about the longitudinal axis of the abutment wall

$$M_{uWallLC4StrI} := 0.9 \cdot DC_{backwall} \cdot \frac{(t_{backwall} - t_{wall})}{2} \ldots$$
$$+ (1.25 \cdot DC_{Sup} + 1.5 \cdot DW_{Sup}) \cdot \left(l_{brtowall} - \frac{t_{wall}}{2}\right) \ldots$$
$$+ 1.5 \cdot P_{EHWall} \cdot \frac{(h_{backwall} + h_{wall})}{3} + 1.75 \cdot P_{LSWall} \cdot \frac{(h_{backwall} + h_{wall})}{2} + 0.5 \cdot TU \cdot h_{wall}$$

$$M_{uWallLC4StrI} = 131.04 \cdot \frac{kip \cdot ft}{ft}$$

Service I

Service I = 1.0DC + 1.0DW + 1.0LL + 1.0BR + 1.0WS + 1.0WL + 1.0EH + 1.0EV + 1.0LS + 1.0TU

Load Case I

Factored vertical force $F_{VWallLC1SerI} := DC_{backwall} + DC_{wall} = 9.29 \cdot \frac{kip}{ft}$

Factored shear force parallel to the transverse axis of the abutment wall $V_{uWallLC1SerI} := P_{EHWall} = 8.55 \cdot \frac{kip}{ft}$

Factored moment about the longitudinal axis of the abutment wall

$$M_{uWallLC1SerI} := DC_{backwall} \cdot \frac{(t_{backwall} - t_{wall})}{2} + P_{EHWall} \cdot \frac{(h_{backwall} + h_{wall})}{3}$$

$$M_{uWallLC1SerI} = 61.28 \cdot \frac{kip \cdot ft}{ft}$$

Load Case III

Factored vertical force

$$F_{VWallLC3SerI} := (DC_{Sup} + DC_{backwall} + DC_{wall}) + DW_{Sup} + R_{LLWallMax} = 21.46 \cdot \frac{kip}{ft}$$

Factored shear force parallel to the transverse axis of the abutment wall $V_{uWallLC3SerI} := P_{EHWall} = 8.55 \cdot \frac{kip}{ft}$

Factored moment about the longitudinal axis of the abutment wall

$$M_{uWallLC3SerI} := DC_{backwall} \cdot \frac{(t_{backwall} - t_{wall})}{2} \ldots$$
$$+ (DC_{Sup} + DW_{Sup} + R_{LLWallMax}) \cdot \left(l_{brtowall} - \frac{t_{wall}}{2}\right) \ldots$$
$$+ P_{EHWall} \cdot \frac{(h_{backwall} + h_{wall})}{3}$$

$$M_{uWallLC3SerI} = 70.41 \cdot \frac{kip \cdot ft}{ft}$$

Load Case IV

Factored vertical force

$$F_{VWallLC4SerI} := \left(DC_{Sup} + DC_{backwall} + DC_{wall}\right) + DW_{Sup}$$

$$F_{VWallLC4SerI} = 15.83 \cdot \frac{kip}{ft}$$

Factored shear force parallel to the transverse axis of the abutment wall

$$V_{uWallLC4SerI} := P_{EHWall} + P_{LSWall} + TU = 10.39 \cdot \frac{kip}{ft}$$

Factored moment about the longitudinal axis of the abutment wall

$$M_{uWallLC4SerI} := DC_{backwall} \cdot \frac{\left(t_{backwall} - t_{wall}\right)}{2} \ldots$$
$$+ \left(DC_{Sup} + DW_{Sup}\right) \cdot \left(l_{brtowall} - \frac{t_{wall}}{2}\right) \ldots$$
$$+ P_{EHWall} \cdot \frac{\left(h_{backwall} + h_{wall}\right)}{3} + P_{LSWall} \cdot \frac{\left(h_{backwall} + h_{wall}\right)}{2} + TU \cdot h_{wall}$$

$$M_{uWallLC4SerI} = 88.15 \cdot \frac{kip \cdot ft}{ft}$$

Summary of Forces and Moments at the Base of the Abutment Wall

Factored vertical force, F_{VWall} (kip/ft)

	Strength I	Service I
LC I	11.61	9.29
LC III	29.86	21.46
LC IV	20.01	15.83

Factored shear force parallel to the transverse axis of the abutment wall, V_{uWall} (kip/ft)

	Strength I	Service I
LC I	12.82	8.55
LC III	12.82	8.55
LC IV	15.70	10.39

Factored moment about the longitudinal axis of the abutment wall, M_{uWall} (kip·ft/ft)

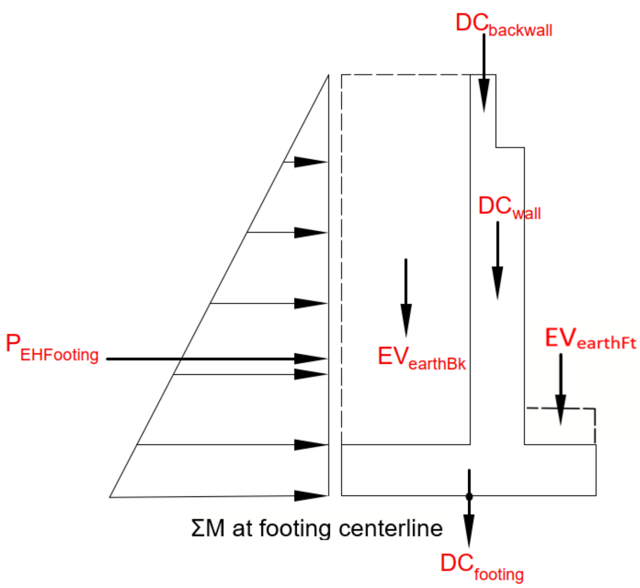
	Strength I	Service I
LC I	92.40	61.28
LC III	106.09	70.41
LC IV	131.04	88.15

Forces and Moments at the Base of the Footing

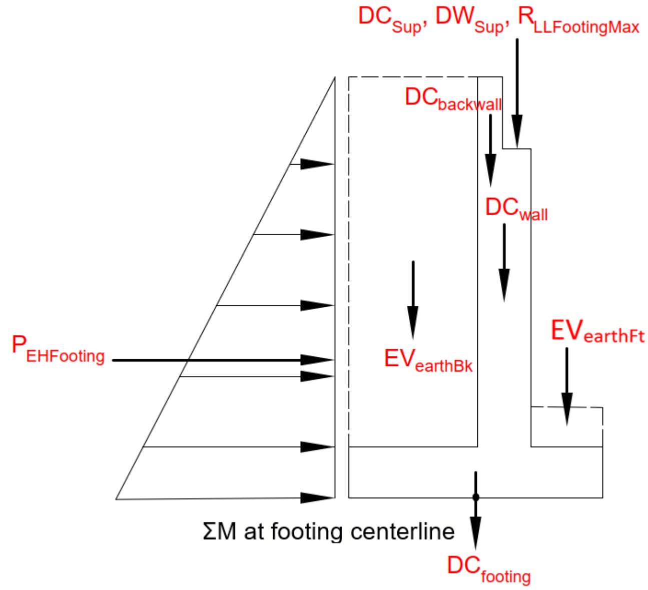
Load Cases I, III, and IV are considered. In addition to all the loads considered for the abutment wall, weight of soil (i.e. the earth load on the footing toe and heel) and live load on the backwall are considered.

The dynamic load allowance is excluded from the live load for foundation components that are entirely below ground level.

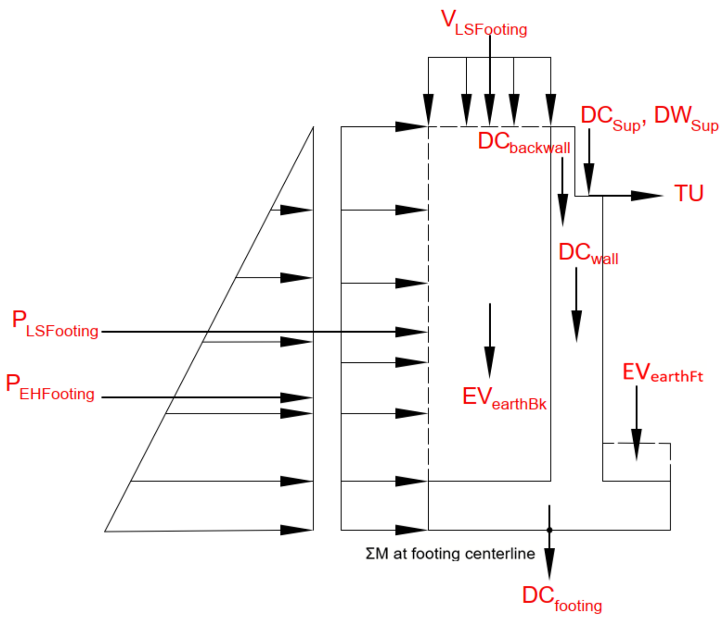
LRFD 3.6.2.1



LC I



LC III



LC IV

Strength I

Strength I = 1.25DC + 1.5DW + 1.75LL + 1.75BR + 1.5EH + 1.35EV + 1.75LS + 0.5TU

Load Case I

Factored vertical force

$$F_{VFtLC1StrI} := 1.25\cdot\left(DC_{backwall} + DC_{wall} + DC_{footing}\right) + 1.35\cdot\left(EV_{earthBk} + EV_{earthFt}\right) = 56.79\cdot\frac{kip}{ft}$$

Factored shear force parallel to the transverse axis of the footing

$$V_{uFtLC1StrI} := 1.5\cdot P_{EHFooting} = 16.59\cdot\frac{kip}{ft}$$

The backfill soil weight reduces the critical moment at the base of the footing. This requires using the minimum load factor of 1.0 for the vertical earth load (EV) instead of the factor 1.35 in the Strength I combination. Similar conditions are applied for the moments calculated about the longitudinal axis of the footing for all the load cases.

LRFD 3.4.1
LFRD Table 3.4.1-2

Factored moment about the longitudinal axis of the footing

$$M_{uFtLC1StrI} := 1.25\cdot DC_{backwall}\cdot\left(l_{heel} + \frac{t_{backwall}}{2} - \frac{B_{footing}}{2}\right) + 1.25DC_{wall}\cdot\left(l_{heel} + \frac{t_{wall}}{2} - \frac{B_{footing}}{2}\right) \ldots$$
$$+ 1.5\cdot P_{EHFooting}\cdot\frac{\left(h_{backwall} + h_{wall} + t_{footing}\right)}{3} + 1.35EV_{earthFt}\cdot\left(\frac{B_{footing}}{2} - \frac{l_{toe}}{2}\right) \ldots$$
$$+ 1.0\cdot EV_{earthBk}\cdot\left(\frac{l_{heel}}{2} - \frac{B_{footing}}{2}\right)$$

$$M_{uFtLC1StrI} = 87.92\cdot\frac{kip\cdot ft}{ft}$$

Load Case III

Factored vertical force

$$F_{VFtLC3StrI} := 1.25\cdot\left(DC_{Sup} + DC_{backwall} + DC_{wall} + DC_{footing}\right) + 1.5DW_{Sup} \ldots$$
$$+ 1.75R_{LLFootingMax} + 1.35\cdot\left(EV_{earthBk} + EV_{earthFt}\right)$$

$$F_{VFtLC3StrI} = 74.75\cdot\frac{kip}{ft}$$

Factored shear force parallel to the transverse axis of the footing

$$V_{uFtLC3StrI} := 1.5\cdot P_{EHFooting} = 16.59\cdot\frac{kip}{ft}$$

Factored moment about the longitudinal axis of the footing

$$M_{uFtLC3StrI} := 1.25\cdot DC_{backwall}\cdot\left(l_{heel} + \frac{t_{backwall}}{2} - \frac{B_{footing}}{2}\right) + 1.25DC_{wall}\cdot\left(l_{heel} + \frac{t_{wall}}{2} - \frac{B_{footing}}{2}\right) \ldots$$
$$+ \left(1.25\cdot DC_{Sup} + 1.5\cdot DW_{Sup} + 1.75\cdot R_{LLFootingMax}\right)\cdot\left(l_{heel} + l_{brtowall} - \frac{B_{footing}}{2}\right) \ldots$$
$$+ 1.5\cdot P_{EHFooting}\cdot\frac{\left(h_{backwall} + h_{wall} + t_{footing}\right)}{3} \ldots$$
$$+ 1.0\cdot EV_{earthBk}\cdot\left(\frac{l_{heel}}{2} - \frac{B_{footing}}{2}\right) + 1.35\cdot EV_{earthFt}\cdot\left(\frac{B_{footing}}{2} - \frac{l_{toe}}{2}\right)$$

$$M_{uFtLC3StrI} = 143.27\cdot\frac{kip\cdot ft}{ft}$$

Load Case IV

Factored vertical force

$$F_{VFtLC4StrI} := 1.25 \cdot \left(DC_{Sup} + DC_{backwall} + DC_{wall} + DC_{footing}\right) + 1.5DW_{Sup} \cdots$$
$$+ 1.35 \cdot \left(EV_{earthFt} + EV_{earthBk}\right) + 1.75V_{LSFooting}$$

$$F_{VFtLC4StrI} = 69.08 \cdot \frac{kip}{ft}$$

Factored shear force parallel to the transverse axis of the footing

$$V_{uFtLC4StrI} := 1.5 \cdot P_{EHFooting} + 1.75P_{LSFooting} + 0.5TU = 19.85 \cdot \frac{kip}{ft}$$

Factored moment about the longitudinal axis of the footing

$$M_{uFtLC4StrI} := 1.25 \cdot DC_{backwall} \cdot \left(l_{heel} + \frac{t_{backwall}}{2} - \frac{B_{footing}}{2}\right) + 1.25DC_{wall} \cdot \left(l_{heel} + \frac{t_{wall}}{2} - \frac{B_{footing}}{2}\right) \cdots$$
$$+ \left(1.25 \cdot DC_{Sup} + 1.5 \cdot DW_{Sup}\right) \cdot \left(l_{heel} + l_{brtowall} - \frac{B_{footing}}{2}\right) \cdots$$
$$+ 1.5 \cdot P_{EHFooting} \cdot \frac{\left(h_{backwall} + h_{wall} + t_{footing}\right)}{3} + 1.75V_{LSFooting} \cdot \left(\frac{l_{heel}}{2} - \frac{B_{footing}}{2}\right) \cdots$$
$$+ 1.75 \cdot P_{LSFooting} \cdot \frac{\left(h_{backwall} + h_{wall} + t_{footing}\right)}{2} + 1.0 \cdot EV_{earthBk} \cdot \left(\frac{l_{heel}}{2} - \frac{B_{footing}}{2}\right) \cdots$$
$$+ 1.35 \cdot EV_{earthFt} \cdot \left(\frac{B_{footing}}{2} - \frac{l_{toe}}{2}\right) + 0.5 \cdot TU \cdot \left(h_{wall} + t_{footing}\right)$$

$$M_{uFtLC4StrI} = 140.34 \cdot \frac{kip \cdot ft}{ft}$$

Service I

Service I = 1.0DC + 1.0DW + 1.0LL + 1.0BR + 1.0WS + 1.0WL + 1.0EH + 1.0EV + 1.0LS + 1.0TU

Load Case I

Factored vertical force

$$F_{VFtLC1SerI} := DC_{backwall} + DC_{wall} + DC_{footing} + EV_{earthBk} + EV_{earthFt} = 43.32 \cdot \frac{kip}{ft}$$

Factored shear force parallel to the transverse axis of the footing

$$V_{uFtLC1SerI} := P_{EHFooting} = 11.06 \cdot \frac{kip}{ft}$$

Factored moment about the longitudinal axis of the footing

$$M_{uFtLC1SerI} := DC_{backwall} \cdot \left(l_{heel} + \frac{t_{backwall}}{2} - \frac{B_{footing}}{2}\right) + DC_{wall} \cdot \left(l_{heel} + \frac{t_{wall}}{2} - \frac{B_{footing}}{2}\right) \cdots$$
$$+ P_{EHFooting} \cdot \frac{\left(h_{backwall} + h_{wall} + t_{footing}\right)}{3} \cdots$$
$$+ EV_{earthBk} \cdot \left(\frac{l_{heel}}{2} - \frac{B_{footing}}{2}\right) + EV_{earthFt} \cdot \left(\frac{B_{footing}}{2} - \frac{l_{toe}}{2}\right)$$

$$M_{uFtLC1SerI} = 32.22 \cdot \frac{kip \cdot ft}{ft}$$

Load Case III

Factored vertical force

$$F_{VFtLC3SerI} := DC_{Sup} + DC_{backwall} + DC_{wall} + DC_{footing} + DW_{Sup} + R_{LLFootingMax} \cdots + \left(EV_{earthFt} + EV_{earthBk}\right)$$

$$F_{VFtLC3SerI} = 55.33 \cdot \frac{kip}{ft}$$

Factored shear force parallel to the transverse axis of the footing

$$V_{uFtLC3SerI} := P_{EHFooting} = 11.06 \cdot \frac{kip}{ft}$$

Factored moment about the longitudinal axis of the footing

$$M_{uFtLC3SerI} := DC_{backwall} \cdot \left(l_{heel} + \frac{t_{backwall}}{2} - \frac{B_{footing}}{2}\right) + DC_{wall} \cdot \left(l_{heel} + \frac{t_{wall}}{2} - \frac{B_{footing}}{2}\right) \cdots + \left(DC_{Sup} + DW_{Sup} + R_{LLFootingMax}\right) \cdot \left(l_{heel} + l_{brtowall} - \frac{B_{footing}}{2}\right) \cdots + P_{EHFooting} \cdot \frac{\left(h_{backwall} + h_{wall} + t_{footing}\right)}{3} \cdots + EV_{earthBk} \cdot \left(\frac{l_{heel}}{2} - \frac{B_{footing}}{2}\right) + EV_{earthFt} \cdot \left(\frac{B_{footing}}{2} - \frac{l_{toe}}{2}\right)$$

$$M_{uFtLC3SerI} = 69.22 \cdot \frac{kip \cdot ft}{ft}$$

Load Case IV

Factored vertical force

$$F_{VFtLC4SerI} := DC_{Sup} + DC_{backwall} + DC_{wall} + DC_{footing} + DW_{Sup} \cdots + EV_{earthFt} + EV_{earthBk} + V_{LSFooting}$$

$$F_{VFtLC4SerI} = 52.09 \cdot \frac{kip}{ft}$$

Factored shear force parallel to the transverse axis of the footing

$$V_{uFtLC4SerI} := P_{EHFooting} + P_{LSFooting} + TU = 13.12 \cdot \frac{kip}{ft}$$

Factored moment about the longitudinal axis of the footing

$$M_{uFtLC4SerI} := DC_{backwall} \cdot \left(l_{heel} + \frac{t_{backwall}}{2} - \frac{B_{footing}}{2}\right) + DC_{wall} \cdot \left(l_{heel} + \frac{t_{wall}}{2} - \frac{B_{footing}}{2}\right) \cdots + \left(DC_{Sup} + DW_{Sup}\right) \cdot \left(l_{heel} + l_{brtowall} - \frac{B_{footing}}{2}\right) + P_{EHFooting} \cdot \frac{\left(h_{backwall} + h_{wall} + t_{footing}\right)}{3} \cdots + EV_{earthBk} \cdot \left(\frac{l_{heel}}{2} - \frac{B_{footing}}{2}\right) + EV_{earthFt} \cdot \left(\frac{B_{footing}}{2} - \frac{l_{toe}}{2}\right) \cdots + V_{LSFooting} \cdot \left(\frac{l_{heel}}{2} - \frac{B_{footing}}{2}\right) + P_{LSFooting} \cdot \frac{\left(h_{backwall} + h_{wall} + t_{footing}\right)}{2} \cdots + TU \cdot \left(h_{wall} + t_{footing}\right)$$

$$M_{uFtLC4SerI} = 71.62 \cdot \frac{kip \cdot ft}{ft}$$

Summary of Forces and Moments at the Base of the Footing

Factored vertical force, F_{VFt} (kip/ft)

	Strength I	Service I
LC I	56.79	43.32
LC III	74.75	55.33
LC IV	69.08	52.09

Factored shear force parallel to the transverse axis of the footing, V_{uFt} (kip/ft)

	Strength I	Service I
LC I	16.59	11.06
LC III	16.59	11.06
LC IV	19.85	13.12

Factored moment about the longitudinal axis of the footing, M_{uFt} (kip·ft/ft)

	Strength I	Service I
LC I	87.92	32.22
LC III	143.27	69.22
LC IV	140.34	71.62

Step 2.6 Geotechnical Design of the Footing

Description

This step presents the geotechnical design of a spread footing considering the following strength and serviceability limit states:

LRFD 10.6.1.1

1. bearing resistance – strength limit state
2. settlement – service limit state
3. sliding resistance – strength limit state
4. load eccentricity (overturning) – strength limit state

Step 2.9 presents the evaluation of structural resistance of the footing (internal stability).

Page **Contents**

Forces and Moments at the Base of the Footing

Step 2.5 presents the load effects at the base of the footing under different load cases and limit states. A summary is presented in the following tables:

Factored vertical force, F_{VFt} (kip/ft)

	Strength I	Service I
LC I	56.79	43.32
LC III	74.75	55.33
LC IV	69.08	52.09

Factored shear force parallel to the transverse axis of the footing, V_{uFt} (kip/ft)

	Strength I	Service I
LC I	16.59	11.06
LC III	16.59	11.06
LC IV	19.85	13.12

Factored moment about the longitudinal axis of the footing, M_{uFt} (kip·ft/ft)

	Strength I	Service I
LC I	87.92	32.22
LC III	143.27	69.22
LC IV	140.34	71.62

Bearing Resistance Check

For eccentrically loaded footings, the use of a reduced effective area is allowed for bearing resistance or settlement calculation. The point of load application shall be at the centroid of the reduced area. **LRFD 10.6.1.3**

Note: As a practice, the average pressure and the values at the toe and heel under different load cases and limit states are provided to the MDOT Geotechnical Service Section for verification.

This example presents the LRFD and MDOT methods.

Load Case I, Strength I

Factored vertical force $$F_{VFtLC1StrI} = 56.79 \cdot \frac{kip}{ft}$$

Factored moment about the longitudinal axis of the footing $$M_{uFtLC1StrI} = 87.92 \cdot \frac{kip \cdot ft}{ft}$$

Eccentricity in the footing width direction $$e_B := \frac{M_{uFtLC1StrI}}{F_{VFtLC1StrI}} = 1.55\ ft$$

<u>LRFD Method</u>

A reduced effective footing width is used for bearing resistance and settlement design. **LRFD 10.6.1.3**

Effective footing width $$B_{eff} := B_{footing} - 2 \cdot e_B = 13.9\ ft$$ **LRFD Eq. 10.6.1.3-1**

Bearing pressure $$q_{bearing\_LC1} := \frac{F_{VFtLC1StrI}}{B_{eff}} = 4.08 \cdot ksf$$

MDOT Method

Average bearing pressure

$$q_{avgLC1} := \frac{F_{VFtLC1StrI}}{B_{footing}} = 3.34 \cdot ksf$$

Bearing pressure at the toe

$$q_{toeLC1} := \frac{F_{VFtLC1StrI}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}}\right) = 5.17 \cdot ksf$$

Bearing pressure at the heel

$$q_{heelLC1} := \frac{F_{VFtLC1StrI}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}}\right) = 1.52 \cdot ksf$$

Load Case III, Strength I

Factored vertical force

$$F_{VFtLC3StrI} = 74.75 \cdot \frac{kip}{ft}$$

Factored moment about the longitudinal axis of the footing

$$M_{uFtLC3StrI} = 143.27 \cdot \frac{kip \cdot ft}{ft}$$

Eccentricity in the footing width direction

$$e_B := \frac{M_{uFtLC3StrI}}{F_{VFtLC3StrI}} = 1.92\ ft$$

LRFD Method

Effective footing width

$$B_{eff} := B_{footing} - 2 \cdot e_B = 13.17\ ft$$

LRFD Eq. 10.6.1.3-1

Bearing pressure

$$q_{bearing\_LC3} := \frac{F_{VFtLC3StrI}}{B_{eff}} = 5.68 \cdot ksf$$

MDOT Method

Average bearing pressure

$$q_{avgLC3} := \frac{F_{VFtLC3StrI}}{B_{footing}} = 4.4 \cdot ksf$$

Bearing pressure at the toe

$$q_{toeLC3} := \frac{F_{VFtLC3StrI}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}}\right) = 7.37 \cdot ksf$$

Bearing pressure at the heel

$$q_{heelLC3} := \frac{F_{VFtLC3StrI}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}}\right) = 1.42 \cdot ksf$$

Load Case IV, Strength I

Factored vertical force

$$F_{VFtLC4StrI} = 69.08 \cdot \frac{kip}{ft}$$

Factored moment about the longitudinal axis of the footing

$$M_{uFtLC4StrI} = 140.34 \cdot \frac{kip \cdot ft}{ft}$$

Eccentricity in the footing width direction

$$e_B := \frac{M_{uFtLC4StrI}}{F_{VFtLC4StrI}} = 2.03\ ft$$

LRFD Method

Effective footing width $B_{eff} := B_{footing} - 2 \cdot e_B = 12.94\ ft$ **LRFD Eq. 10.6.1.3-1**

Bearing pressure $q_{bearing\_LC4} := \frac{F_{VFtLC4StrI}}{B_{eff}} = 5.34 \cdot ksf$

MDOT Method

Average bearing pressure $q_{avgLC4} := \frac{F_{VFtLC4StrI}}{B_{footing}} = 4.06 \cdot ksf$

Bearing pressure at the toe $q_{toeLC4} := \frac{F_{VFtLC4StrI}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}}\right) = 6.98 \cdot ksf$

Bearing pressure at the heel $q_{heelLC4} := \frac{F_{VFtLC4StrI}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}}\right) = 1.15 \cdot ksf$

Load Case I, Service I

Factored vertical force $F_{VFtLC1SerI} = 43.32 \cdot \frac{kip}{ft}$

Factored moment about the longitudinal axis of the footing $M_{uFtLC1SerI} = 32.22 \cdot \frac{kip \cdot ft}{ft}$

Eccentricity in the footing width direction $e_B := \frac{M_{uFtLC1SerI}}{F_{VFtLC1SerI}} = 0.74\ ft$

LRFD Method

Effective footing width $B_{eff} := B_{footing} - 2 \cdot e_B = 15.51\ ft$ **LRFD Eq. 10.6.1.3-1**

Bearing pressure $q_{bearing\_LC1SerI} := \frac{F_{VFtLC1SerI}}{B_{eff}} = 2.79 \cdot ksf$

MDOT Method

Average bearing pressure $q_{avgLC1SerI} := \frac{F_{VFtLC1SerI}}{B_{footing}} = 2.55 \cdot ksf$

Bearing pressure at the toe $q_{toeLC1SerI} := \frac{F_{VFtLC1SerI}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}}\right) = 3.22 \cdot ksf$

Bearing pressure at the heel $q_{heelLC1SerI} := \frac{F_{VFtLC1SerI}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}}\right) = 1.88 \cdot ksf$

Load Case III, Service I

Factored vertical force $F_{VFtLC3SerI} = 55.33 \cdot \frac{kip}{ft}$

Factored moment about the longitudinal axis of the footing $M_{uFtLC3SerI} = 69.22 \cdot \frac{kip \cdot ft}{ft}$

Eccentricity in the footing width direction $e_B := \frac{M_{uFtLC3SerI}}{F_{VFtLC3SerI}} = 1.25\ ft$

LRFD Method

Effective footing width $B_{eff} := B_{footing} - 2 \cdot e_B = 14.5\ ft$ **LRFD Eq. 10.6.1.3-1**

Bearing pressure $q_{bearing\_LC3SerI} := \frac{F_{VFtLC3SerI}}{B_{eff}} = 3.82 \cdot ksf$

MDOT Method

Average bearing pressure $q_{avgLC3SerI} := \frac{F_{VFtLC3SerI}}{B_{footing}} = 3.25 \cdot ksf$

Bearing pressure at the toe $q_{toeLC3SerI} := \frac{F_{VFtLC3SerI}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}}\right) = 4.69 \cdot ksf$

Bearing pressure at the heel $q_{heelLC3SerI} := \frac{F_{VFtLC3SerI}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}}\right) = 1.82 \cdot ksf$

Load Case IV, Service I

Factored vertical force $F_{VFtLC4SerI} = 52.09 \cdot \frac{kip}{ft}$

Factored moment about the longitudinal axis of the footing $M_{uFtLC4SerI} = 71.62 \cdot \frac{kip \cdot ft}{ft}$

Eccentricity in the footing width direction $e_B := \frac{M_{uFtLC4SerI}}{F_{VFtLC4SerI}} = 1.37\ ft$

LRFD Method

Effective footing width $B_{eff} := B_{footing} - 2 \cdot e_B = 14.25\ ft$ **LRFD Eq. 10.6.1.3-1**

Bearing pressure $q_{bearing\_LC4SerI} := \frac{F_{VFtLC4SerI}}{B_{eff}} = 3.66 \cdot ksf$

MDOT Method

Average bearing pressure $q_{avgLC4SerI} := \frac{F_{VFtLC4SerI}}{B_{footing}} = 3.06 \cdot ksf$

Bearing pressure at the toe $q_{toeLC4SerI} := \frac{F_{VFtLC4SerI}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}}\right) = 4.55 \cdot ksf$

Bearing pressure at the heel $q_{heelLC4SerI} := \frac{F_{VFtLC4SerI}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}}\right) = 1.58 \cdot ksf$

Summary

LRFD Method

The controlling bearing pressure under strength limit states

$$q_b := \max\left(q_{bearing\_LC1}, q_{bearing\_LC3}, q_{bearing\_LC4}\right) = 5.68 \cdot ksf$$

The controlling bearing pressure needs to be checked with the factored bearing resistance of the soil provided by the Geotechnical Service Section.

MDOT Method

A summary of bearing pressure values (in psf) are shown in the following table:

	Toe (Service I)	Avg (Serivce I)	Heel (Service I)	Toe (Strength I)	Avg (Strength I)	Heel (Strength I)
LC I	3217	2549	1880	5166	3341	1516
LC III	4692	3254	1817	7371	4397	1422
LC IV	4551	3064	1577	6977	4064	1150

This table is provided to the Geotechnical Service Section for the verification of bearing resistance and settlement limits. If the bearing pressure exceeds the bearing strength of the soil, the size of the footing needs to be increased. See BDM 7.03.02.G for more information.

Settlement Check

The Geotechnical Service Section uses the controlling bearing pressure from the service limit state to check if the foundation total settlement is less than 1.5 in., the allowable limit. **BDM 7.03.02G 2b**

For LRFD method, the controlling bearing pressure for settlement analysis is

$$q_{b\_settlement} := \max\left(q_{bearing\_LC1SerI}, q_{bearing\_LC3SerI}, q_{bearing\_LC4SerI}\right) = 3.82 \cdot ksf$$

The Geotechnical Service Section uses this controlling bearing pressure to calculate the foundation total settlement.

For MDOT method, the bearing pressures under service limit state are provided to the Geotechnical Service Section to calculate the settlement.

Note: Besides the total settlement, considerations should be given to prevent the differential settlement between the abutments and pier from exceeding the tolerable differential settlement limit. Differential settlement limits are given in the *Steel Plate Girder Design Example.*

Sliding Resistance Check

Spread footings must be designed to resist lateral loads without sliding. The sliding resistance of a footing on cohesionless soil is a function of the normal force and the interface friction between the foundation and the soil. **LRFD 10.6.3.4**

The Geotechnical Service Section should provide a coefficient of sliding resistance (μ) for a design. MDOT typically uses a sliding resistance coefficient of 0.5 for cast-in-place concrete footings. Consult the Geotechnical Service Section to identify the most suitable coefficient for a specific design.

Coefficient of sliding resistance $\mu := 0.5$

The strength limit states are used for this check. Since the resistance is proportional to the vertical loads, the following conditions are used:

- Minimum load factors are used for all vertical loads.
- Maximum load factors are used for the loads that contribute to horizontal sliding forces.
- Since DW is the future wearing surface load, it is excluded from all load combinations.

Load Case I

Factored shear force parallel to the transverse axis of the footing $\quad V_{uFtLC1StrI} = 16.59 \cdot \frac{kip}{ft}$

Factored sliding force $\quad V_{sliding} := V_{uFtLC1StrI} = 16.59 \cdot \frac{kip}{ft}$

Minimum vertical load

$$F_{VFtLC1StrIMin} := 0.9 \cdot \left(DC_{backwall} + DC_{wall} + DC_{footing}\right) + 1.0 \cdot \left(EV_{earthBk} + EV_{earthFt}\right) = 41.63 \cdot \frac{kip}{ft}$$

Resistance factor for sliding $\quad \phi_{\tau} := 0.8 \quad$ **BDM 7.03.02.F, LRFD Table 10.5.5.5.2-1**

Sliding resistance $\quad V_{resistance} := \phi_{\tau} \cdot \mu \cdot F_{VFtLC1StrIMin} = 16.65 \cdot \frac{kip}{ft}$

Check if $V_{resistance} > V_{sliding}$ $\quad Check := if\left(V_{resistance} > V_{sliding}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Load Case III

Factored shear force parallel to the transverse axis of the footing $\quad V_{uFtLC3StrI} = 16.59 \cdot \frac{kip}{ft}$

Factored sliding force $\quad V_{sliding} := V_{uFtLC3StrI} = 16.59 \cdot \frac{kip}{ft}$

When calculating the minimum vertical force for sliding and eccentric load limitation check, the live load on the superstructure is excluded to develop a conservative design.

Minimum vertical load without the live load

$$F_{VFtLC3StrIMin\_noLL} := 0.9 \cdot \left(DC_{Sup} + DC_{backwall} + DC_{wall} + DC_{footing}\right) \cdots + 1.0 \cdot \left(EV_{earthBk} + EV_{earthFt}\right)$$

$$F_{VFtLC3StrIMin\_noLL} = 46.72 \cdot \frac{kip}{ft}$$

Resistance factor for sliding $\quad \phi_{\tau} := 0.8 \quad$ **BDM 7.03.02.F, LRFD Table 10.5.5.5.2-1**

Sliding resistance $\quad V_{resistance} := \phi_{\tau} \cdot \mu \cdot F_{VFtLC3StrIMin\_noLL} = 18.69 \cdot \frac{kip}{ft}$

Check if $V_{resistance} > V_{sliding}$ $\quad Check := if\left(V_{resistance} > V_{sliding}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Load Case IV

There are two cases that need to be considered: without and with the live load surcharge.

Without the live load surcharge:

Factored shear force parallel to the transverse axis of the footing $\quad V_{uFtLC4StrI} = 19.85 \cdot \frac{kip}{ft}$

Factored sliding force without the live load surcharge

$$V_{sliding} := V_{uFtLC4StrI} - 1.75P_{LSFooting} = 16.73 \cdot \frac{kip}{ft}$$

Minimum vertical load without the live load surcharge

$$F_{VFtLC4StrIMin\_noLS} := 0.9 \cdot \left(DC_{Sup} + DC_{backwall} + DC_{wall} + DC_{footing}\right) \ldots + 1.0 \cdot \left(EV_{earthBk} + EV_{earthFt}\right)$$

$$F_{VFtLC4StrIMin\_noLS} = 46.72 \cdot \frac{kip}{ft}$$

Sliding resistance

$$V_{resistance} := \phi_{\tau} \cdot \mu \cdot F_{VFtLC4StrIMin\_noLS} = 18.69 \cdot \frac{kip}{ft}$$

Check if $V_{resistance} > V_{sliding}$

$$Check := if\left(V_{resistance} > V_{sliding}, "OK", "Not\ OK"\right) = "OK"$$

With the live load surcharge:

Factored shear force parallel to the transverse axis of the footing

$$V_{uFtLC4StrI} = 19.85 \cdot \frac{kip}{ft}$$

Factored sliding force

$$V_{sliding} := V_{uFtLC4StrI} = 19.85 \cdot \frac{kip}{ft}$$

Minimum vertical load with the live load surcharge

$$F_{VFtLC4StrIMin} := 0.9 \cdot \left(DC_{Sup} + DC_{backwall} + DC_{wall} + DC_{footing}\right) \ldots + 1.0 \cdot \left(EV_{earthBk} + EV_{earthFt}\right) + 1.75V_{LSFooting}$$

$$F_{VFtLC4StrIMin} = 50.61 \cdot \frac{kip}{ft}$$

Sliding resistance

$$V_{resistance} := \phi_{\tau} \cdot \mu \cdot F_{VFtLC4StrIMin} = 20.24 \cdot \frac{kip}{ft}$$

Check if $V_{resistance} > V_{sliding}$

$$Check := if\left(V_{resistance} > V_{sliding}, "OK", "Not\ OK"\right) = "OK"$$

Eccentric Load Limitation (Overturning) Check

The eccentricity of loading at the strength limit state, evaluated based on factored loads, shall not exceed one-third of the corresponding dimension of footing on soils for stability. **LRFD 10.6.3.3**

The eccentricity in the abutment length direction is not a concern. The following calculations presents the evaluation of the eccentricity in the abutment width direction for Strength I limit state:

Load Case I

Minimum vertical force

$$F_{VFtLC1StrIMin} = 41.63 \cdot \frac{kip}{ft}$$

Moment about the longitudinal axis of the footing

$$M_{uFtLC1StrI} = 87.92 \cdot \frac{kip \cdot ft}{ft}$$

Eccentricity in the footing width direction measured from the centerline

$$e_B := \frac{M_{uFtLC1StrI}}{F_{VFtLC1StrIMin}} = 2.11\ ft$$

1/6 of footing width

$$\frac{B_{footing}}{6} = 2.83 \cdot ft$$

Check if the eccentric load limitation is satisfied

$$Check := if\left(e_B < \frac{B_{footing}}{6}, "OK", "Not\ OK"\right) = "OK"$$

Load Case III

There are two cases that need to be considered: without and with the live load

Without the live load:

Minimum vertical force without the live load $F_{VFtLC3StrIMin\_noLL} = 46.72 \cdot \frac{kip}{ft}$

Moment about the longitudinal axis of the footing (with the live load) $M_{uFtLC3StrI} = 143.27 \cdot \frac{kip \cdot ft}{ft}$

Moment about the longitudinal axis of the footing (without the live load)

$$M_{uFtLC3StrI\_noLL} := M_{uFtLC3StrI} - (1.75 \cdot R_{LLFootingMax}) \cdot \left(l_{heel} + l_{brtowall} - \frac{B_{footing}}{2} \right) = 113.82 \cdot \frac{kip \cdot ft}{ft}$$

Eccentricity in the footing width direction $e_B := \frac{M_{uFtLC3StrI\_noLL}}{F_{VFtLC3StrIMin\_noLL}} = 2.44\ ft$

Check if the eccentric load limitation is satisfied $Check := if\left(e_B < \frac{B_{footing}}{6}, "OK", "Not\ OK" \right) = "OK"$

With the live load:

Minimum vertical force with the live load $F_{VFtLC3StrIMin} := F_{VFtLC3StrIMin\_noLL} + 1.75 R_{LLFootingMax}$

$F_{VFtLC3StrIMin} = 56.27 \cdot \frac{kip}{ft}$

Moment about the longitudinal axis of the footing (with the live load) $M_{uFtLC3StrI} = 143.27 \cdot \frac{kip \cdot ft}{ft}$

Eccentricity in the footing width direction $e_B := \frac{M_{uFtLC3StrI}}{F_{VFtLC3StrIMin}} = 2.55\ ft$

Check if the eccentric load limitation is satisfied $Check := if\left(e_B < \frac{B_{footing}}{6}, "OK", "Not\ OK" \right) = "OK"$

Load Case IV

There are two cases that need to be considered: without and with the live load surcharge.

Without the live load surcharge:

Minimum vertical force $F_{VFtLC4StrIMin\_noLS} = 46.72 \cdot \frac{kip}{ft}$

Moment about the longitudinal axis of the footing (with the live load surcharge) $M_{uFtLC4StrI} = 140.34 \cdot \frac{kip \cdot ft}{ft}$

Moment about the longitudinal axis of the footing (without the live load surcharge)

$$M_{uFtLC4StrI\_noLS} := M_{uFtLC4StrI} - 1.75V_{LSFooting} \cdot \left(\frac{l_{heel}}{2} - \frac{B_{footing}}{2} \right) \dots$$
$$+ -1.75 \cdot P_{LSFooting} \cdot \frac{\left(h_{backwall} + h_{wall} + t_{footing}\right)}{2}$$

$$M_{uFtLC4StrI\_noLS} = 116.67 \cdot \frac{kip \cdot ft}{ft}$$

Eccentricity in the footing width direction $e_B := \dfrac{M_{uFtLC4StrI\_noLS}}{F_{VFtLC4StrIMin\_noLS}} = 2.5\ ft$

Check if the eccentric load limitation is satisfied $Check := if\left(e_B < \dfrac{B_{footing}}{6}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

With the live load surcharge:

Minimum vertical force $F_{VFtLC4StrIMin} = 50.61 \cdot \dfrac{kip}{ft}$

Moment about the longitudinal axis of the footing $M_{uFtLC4StrI} = 140.34 \cdot \dfrac{kip \cdot ft}{ft}$

Eccentricity in the footing width direction $e_B := \dfrac{M_{uFtLC4StrI}}{F_{VFtLC4StrIMin}} = 2.77\ ft$

Check if the eccentric load limitation is satisfied $Check := if\left(e_B < \dfrac{B_{footing}}{6}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Step 2.7 Backwall Design

Description

This step presents the design of the backwall.

Page Contents

Forces and Moments at the Base of the Backwall

Step 2.5 presents the load effects at the base of the backwall under different load cases and limit states. A summary is presented in the following tables:

Factored vertical force, F_{VBw} (kip/ft)

	Strength I	Service I
LC I	1.20	-
LC III	1.20	-
LC IV	1.20	0.96

Factored shear force parallel to the transverse axis of the backwall, V_{uBw} (kip/ft)

	Strength I	Service I
LC I	0.49	-
LC III	0.49	-
LC IV	1.02	0.63

Factored moment about the longitudinal axis of the backwall, M_{uBw} (kip·ft/ft)

	Strength I	Service I
LC I	0.69	-
LC III	0.69	-
LC IV	1.83	1.11

Design for Flexure

According to the loads in the summary tables, Load Case IV under Strength I limit state is the governing load case for the flexural design.

Moment demand at the base of the backwall $\quad M_{DemandBackwall} := M_{uBwLC4StrI} = 1.83 \cdot \frac{kip \cdot ft}{ft}$

Flexural Resistance

LRFD 5.6.3.2

The design procedure consists of calculating the reinforcing steel area required to satisfy the moment demand and checking the selected steel area against the requirements and limitations for developing an adequate moment capacity, controlling crack width, and managing shrinkage and temperature stresses.

As a trial, select No. 6 bars. $\quad bar := 6$

Nominal diameter of a reinforcing steel bar $\quad d_{bar} := Dia(bar) = 0.75 \cdot in$

Cross-section area of the bar $\quad A_{bar} := Area(bar) = 0.44 \cdot in^2$

The spacing shall not exceed 3 times the component thickness for members at most 18 in. thick. **LRFD 5.10.6**

Backwall thickness $\quad t_{backwall} = 18 \cdot in$

Selected bar spacing $\quad s_{bar} := 18 \cdot in$

Area of reinforcing steel provided in a 1-ft wide section
$$A_{sProvided} := \frac{A_{bar} \cdot 12in}{s_{bar}} = 0.29 \cdot in^2$$

Effective depth
$$d_e := t_{backwall} - Cover_{bw} = 15 \cdot in$$

Resistance factor for flexure
$$\phi_f := 0.9$$
LRFD 5.5.4.2

A 1-ft wide strip is selected for the design.

Width of the compression face of the section
$$b := 12in$$
LRFD 5.6.2.2

Stress block factor
$$\beta_1 := \min\left[\max\left[0.85 - 0.05 \cdot \left(\frac{f_c - 4ksi}{ksi}\right), 0.65\right], 0.85\right] = 0.85$$

Solve the following equation of A_s to calculate the required area of steel to satisfy the moment demand. Use an assumed initial A_s value to solve the equation.

Initial assumption
$$A_s := 0.3in^2$$

Given
$$M_{DemandBackwall} \cdot ft = \phi_f \cdot A_s \cdot f_y \cdot \left[d_e - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b}\right)\right]$$

Required area of steel
$$A_{s.req} := Find(A_s) = 0.03 \cdot in^2$$

Check if $A_{sProvided} > A_{sRequired}$
$$Check := if(A_{sProvided} > A_{s.req}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$$

Moment capacity of the section with the provided steel
$$M_{CapacityBackwall} := \phi_f \cdot A_{sProvided} \cdot f_y \cdot \frac{\left[d_e - \frac{1}{2} \cdot \left(\frac{A_{sProvided} \cdot f_y}{0.85 \cdot f_c \cdot b}\right)\right]}{ft}$$

$$M_{CapacityBackwall} = 19.42 \cdot \frac{kip \cdot ft}{ft}$$

Distance from the extreme compression fiber to the neutral axis
$$c := \frac{A_{sProvided} \cdot f_y}{0.85 \cdot f_c \cdot \beta_1 \cdot b} = 0.68 \cdot in$$

Check the validity of assumption, $f_s = f_y$
$$Check\_f_s := if\left(\frac{c}{d_e} < 0.6, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Limits for Reinforcement

LRFD 5.6.3.3

The tensile reinforcement provided must be adequate to develop a factored flexural resistance at least equal to the lesser of the cracking moment or 1.33 times the factored moment from the applicable strength limit state load combinations.

Flexural cracking variability factor
$$\gamma_1 := 1.6$$
For concrete structures that are not precast segmental

Ratio of specified minimum yield strength to ultimate tensile strength of the nonprestressed reinforcement
$$\gamma_3 := 0.67$$
For ASTM615 grade 60 reinforcement

Section modulus $S_c := \frac{1}{6} \cdot b \cdot t_{backwall}^2 = 648 \cdot in^3$

Cracking moment $M_{cr} := \frac{\gamma_3 \cdot \gamma_1 \cdot f_r \cdot S_c}{ft} = 24.06 \cdot \frac{kip \cdot ft}{ft}$

1.33 times the factored moment demand $1.33 \cdot M_{DemandBackwall} = 2.43 \cdot \frac{kip \cdot ft}{ft}$

Factored moment to satisfy the minimum reinforcement requirement $M_{req} := \min(1.33 M_{DemandBackwall}, M_{cr}) = 2.43 \cdot \frac{kip \cdot ft}{ft}$

Check the adequacy of the section capacity $Check := \text{if}(M_{CapacityBackwall} > M_{req}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Control of Cracking by Distribution of Reinforcement

LRFD 5.6.7

Limiting the width of expected cracks under service conditions extends the service life. The width of potential cracks can be minimized through proper placement of the reinforcement. The check for crack control requires that the actual stress in the reinforcement should not exceed the service limit state stress.

The spacing requirement for the mild steel reinforcement in the layer closest to the tension face $s \le \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$ **LRFD Eq. 5.6.7-1**

Exposure factor for the Class 1 exposure condition $\gamma_e := 1.00$

Distance from extreme tension fiber to the center of the closest flexural reinforcement $d_c := Cover_{bw} = 3 \cdot in$

Ratio of flexural strain at the extreme tension face to the strain at the centroid of the reinforcement layer closest to the tension face $\beta_s := 1 + \frac{d_c}{0.7(t_{backwall} - d_c)} = 1.29$

The position of the cross-section's neutral axis is determined through an iterative process to calculate the actual stress in the reinforcement. This process starts with an assumed position of the neutral axis as shown below.

Assumed distance from the extreme compression fiber to the neutral axis $x := 3 \cdot in$

Given $\frac{1}{2} \cdot b \cdot x^2 = \frac{E_s}{E_c} \cdot A_{sProvided} \cdot (d_e - x)$

Position of the neutral axis $x_{na} := \text{Find}(x) = 2.234 \cdot in$

Tensile force in the reinforcing steel due to service limit state moment $T_s := \frac{M_{uBackwallSerI}}{d_e - \frac{x_{na}}{3}} \cdot ft = 0.9 \cdot kip$

Stress in the reinforcing steel due to service limit state moment $f_{ss1} := \frac{T_s}{A_{sProvided}} = 3.19 \cdot ksi$

f_{ss} (not to exceed $0.6 f_y$) $f_{ss} := \min(f_{ss1}, 0.6 f_y) = 3.19 \cdot ksi$

Required reinforcement spacing

$$s_{barRequired} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{ss}} - 2 \cdot d_c = 164.79 \cdot in$$

Check if the spacing provided < the required spacing

$$Check := if\left(s_{bar} < s_{barRequired}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Shrinkage and Temperature Reinforcement

The following calculations check the adequacy of the flexural reinforcing steel to control shrinkage and temperature stresses in the backwall:

For bars, the area of reinforcing steel per foot, on each face and in each direction, shall satisfy

$$A_S \geq \frac{1.3bh}{2(b+h)f_y}$$

LRFD 5.10.6

and

$$0.11in^2 \leq A_S \leq 0.6in^2$$

Minimum area of shrinkage and temperature reinforcement

$$A_{shrink.temp} := \min\left[\left[\begin{array}{c} \left(0.60 \frac{in^2}{ft}\right) \\ \max\left[\left[\begin{array}{c} \left(0.11 \frac{in^2}{ft}\right) \\ \left[\frac{1.3 \cdot h_{backwall} \cdot t_{backwall} \cdot \frac{kip}{in \cdot ft}}{2\left(h_{backwall} + t_{backwall}\right) \cdot f_y}\right] \end{array}\right]\right] \end{array}\right]\right] \cdot ft = 0.14 \cdot in^2$$

Check if the provided area of steel > the required area of shrinkage and temperature steel

$$Check := if\left(A_{sProvided} > A_{shrink.temp}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Design for Shear

According to the loads in the summary tables, Load Case IV under Strength I limit state is the governing load case for the shear design.

The maximum factored shear force at the base of the backwall

$$V_{uBwLC4StrI} = 1.02 \cdot \frac{kip}{ft}$$

Effective width of the section

$$b_v := b = 12 \cdot in$$

Depth of the equivalent rectangular stress block

$$a := \frac{A_{sProvided} \cdot f_y}{0.85 \cdot f_c \cdot b} = 0.58 \cdot in$$

Effective shear depth

$$d_v := \max\left(d_e - \frac{a}{2}, 0.9 \cdot d_e, 0.72 \cdot t_{backwall}\right) = 14.71 \cdot in$$

LRFD 5.7.2.8

Note: Since there is no transverse reinforcement in the backwall and the overall depth of the backwall is greater than 16 in., the simplified procedure in LRFD 5.7.3.4.1 cannot be used. The general procedure outlined in LRFD 5.7.3.4.2 is used for the design of backwall shear capacity.

The factored N_u, V_u, and M_u are calculated at the critical section for shear, which is located at a distance d_v from the base of the backwall.

Factored axial force at the critical section (use negative if compression)

$$N_{uBackwallShear} := -\left[1.25\cdot\left(DC_{backwall} - d_v\cdot t_{backwall}\cdot W_c\right)\right]$$

$$N_{uBackwallShear} = -0.85\cdot\frac{kip}{ft}$$

Lateral earth load at the critical section

$$P_{EHBackwallShear} := \frac{1}{2}\cdot k_a\cdot\gamma_s\cdot\left(h_{backwall} - d_v\right)\cdot\left(h_{backwall} - d_v\right)$$

$$P_{EHBackwallShear} = 0.16\cdot\frac{kip}{ft}$$

Load at the critical section due to live load surcharge

$$P_{LSBackwallShear} := k_a\cdot\gamma_s\cdot h_{eq}\cdot\left(h_{backwall} - d_v\right) = 0.22\cdot\frac{kip}{ft}$$

Factored shear force (demand) at the critical section

$$V_{uBackwallShear} := 1.5\cdot P_{EHBackwallShear} + 1.75\cdot P_{LSBackwallShear}$$

$$V_{uBackwallShear} = 0.63\cdot\frac{kip}{ft}$$

Factored moment at the critical section

$$M_{uBackwallShear} := 1.5\cdot P_{EHBackwallShear}\cdot\frac{\left(h_{backwall} - d_v\right)}{3} \ldots$$
$$+ 1.75\cdot P_{LSBackwallShear}\cdot\frac{\left(h_{backwall} - d_v\right)}{2}$$

$$M_{uBackwallShear} = 0.82\cdot\frac{kip\cdot ft}{ft}$$

Check M_u since it cannot be taken less than $V_u d_v$

$$M_{uWallShear} := \max\left(M_{uBackwallShear}, V_{uBackwallShear}\cdot d_v\right)$$

$$M_{uWallShear} = 0.82\cdot\frac{kip\cdot ft}{ft}$$

Net longitudinal tensile strain in the section at the centroid of the tension reinforcement

$$\varepsilon_s := \frac{\left(\frac{M_{uBackwallShear}}{d_v} + 0.5\cdot N_{uBackwallShear} + V_{uBackwallShear}\right)}{E_s\cdot\frac{A_{sProvided}}{ft}}$$

$$\varepsilon_s = 1.03\times 10^{-4}$$ **LRFD Eq. 5.7.3.4.2-4**

Crack spacing parameter

$$s_x := d_v = 1.23\,ft$$

Maximum aggregate size (in.)

$$a_g := 1.5$$ **MDOT Standard Specifications for Construction Table 902-1**

Crack spacing parameter as influenced by the maximum aggregate size

$$s_{xe} := \min\left[\begin{bmatrix} (80in) \\ \max\left[\begin{bmatrix} (12in) \\ \left(s_x\cdot\frac{1.38}{a_g + 0.63}\right)\end{bmatrix}\right]\end{bmatrix}\right] = 12\cdot in$$ **LRFD Eq. 5.7.3.4.2-7**

Factor indicating the ability of diagonally cracked concrete to transmit tension and shear

$$\beta := \frac{4.8}{(1 + 750 \cdot \varepsilon_s)} \cdot \frac{51}{\left(39 + \frac{s_{xe}}{in}\right)} = 4.46$$ **LRFD Eq. 5.7.3.4.2-2**

Nominal shear resistance of concrete, V_n, is calculated as follows:

$$V_{c1} := 0.0316 \cdot \beta \cdot \sqrt{f_c \cdot ksi} \cdot b \cdot d_e = 43.9 \cdot kip$$ **LRFD Eq. 5.7.3.3-3**

$$V_{c2} := 0.25 f_c \cdot b \cdot d_e = 135 \cdot kip$$ **LRFD Eq. 5.7.3.3-2**

$$V_n := \min(V_{c1}, V_{c2}) = 43.9 \cdot kip$$

Resistance factor for shear $\phi_v := 0.9$ **LRFD 5.5.4.2**

Factored shear resistance (capacity) $V_r := \phi_v \cdot V_n = 39.51 \cdot kip$

Check if the shear capacity is greater than the demand

$$Check := if\left(\frac{V_r}{ft} > V_{uBackwallShear}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Shrinkage and Temperature Reinforcement Design

The following calculations check the required amount of reinforcing steel in the secondary direction to control shrinkage and temperature stresses in the backwall.

The reinforcement at the front face of the backwall and the horizontal reinforcement at the interior should satisfy the shrinkage and temperature reinforcement requirements. **LRFD 5.10.6**

The spacing of reinforcement shall not exceed 3 times the component thickness for members at most 18 in. thick. **LRFD 5.10.6**

Note: MDOT practice is to use No. 6 @ 18 in. maximum spacing. **BDG 6.20.03A**

Select No. 6 bars. $bar := 6$

Nominal diameter of a reinforcing steel bar $d_{bST} := Dia(bar) = 0.75 \cdot in$

Cross-section area of the bar $A_{barST} := Area(bar) = 0.44 \cdot in^2$

Spacing of bars $s_{barST} := 18 \cdot in$

Horizontal reinforcing steel area provided in the section

$$A_{sProvidedST} := \frac{A_{barST} \cdot 12in}{s_{barST}} = 0.29 \cdot in^2$$

The required minimum shrinkage and temperature reinforcement area at the backwall was previously calculated during the design of flexural reinforcement.

Required shrinkage and temperature steel area $A_{shrink.temp} = 0.14 \cdot in^2$

Check if the provided steel area > the required area of shrinkage and temperature steel

$$Check := if(A_{sProvidedST} > A_{shrink.temp}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$$

The backwall design presented in this step provides the following details:

- No. 6 bars at 18.0 in. spacing ($A_s = 0.29$ in.^{2}/ft) as the back face flexural reinforcement.
- No. 6 bars at 18.0 in. spacing ($A_s = 0.29$ in.^{2}/ft) as the front face vertical shrinkage and temperature reinforcement.
- No. 6 bars at 18.0 in. spacing ($A_s = 0.29$ in.^{2}/ft) as the front and back face horizontal shrinkage and temperature reinforcement.

Step 2.8 Abutment Wall Design

Description

This step presents the design of the abutment wall.

Page Contents

Forces and Moments at the Base of the Abutment Wall

Step 2.5 presents the load effects at the base of the abutment wall under different load cases and limit states. A summary is presented in the following tables:

Factored vertical force, F_{VWall} (kip/ft)

	Strength I	Service I
LC I	11.61	9.29
LC III	29.86	21.46
LC IV	20.01	15.83

Factored shear force parallel to the transverse axis of the abutment wall, V_{uWall} (kip/ft)

	Strength I	Service I
LC I	12.82	8.55
LC III	12.82	8.55
LC IV	15.70	10.39

Factored moment about the longitudinal axis of the abutment wall, M_{uWall} (kip·ft/ft)

	Strength I	Service I
LC I	92.40	61.28
LC III	106.09	70.41
LC IV	131.04	88.15

Design for Flexure

According to the loads in the summary tables, Load Case IV under Strength I limit state is the governing load case for the flexural design.

Moment demand at the base of the wall $M_{DemandWall} := M_{uWallLC4StrI} = 131.04 \cdot \frac{kip \cdot ft}{ft}$

Flexural Resistance

LRFD 5.6.3.2

The design procedure consists of calculating the reinforcing steel area required to satisfy the moment demand and checking the selected steel area against the requirements and limitations for developing an adequate moment capacity, controlling crack width, and managing shrinkage and temperature stresses.

As a trial, select No. 9 bars. $bar := 9$

Nominal diameter of a reinforcing steel bar $d_{bar} := Dia(bar) = 1.13 \cdot in$

Cross-section area of the bar $A_{bar} := Area(bar) = 1 \cdot in^2$

The spacing of reinforcement shall not exceed 12 in. when the thickness of walls is greater than 18 in.

LRFD 5.10.6

Wall thickness $t_{wall} = 38 \cdot in$

Initial assumption for the spacing of bars $s_{bar} := 12 \cdot in$

Area of reinforcing steel provided in a 1-ft wide section $A_{sProvided} := \frac{A_{bar} \cdot 12in}{s_{bar}} = 1 \cdot in^2$

Effective depth $d_e := t_{wall} - Cover_{wall} = 35 \cdot in$

Resistance factor for flexure $\phi_f := 0.9$

LRFD 5.5.4.2

A 1-ft wide strip is selected for the design.

Width of the compression face of the section $b := 12in$

Stress block factor $\beta_1 := \min\left[\max\left[0.85 - 0.05 \cdot \left(\frac{f_c - 4ksi}{ksi}\right), 0.65\right], 0.85\right] = 0.85$

LRFD 5.6.2.2

Solve the following equation of A_s to calculate the required area of steel to satisfy the moment demand. Use an assumed initial A_s value to solve the equation.

Initial assumption $A_s := 1in^2$

Given $M_{DemandWall} \cdot ft = \phi_f \cdot A_s \cdot f_y \cdot \left[d_e - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b}\right)\right]$

LRFD 5.6.3.2

Required area of steel $A_{sRequired} := Find(A_s) = 0.85 \cdot in^2$

Check if $A_{sProvided} > A_{sRequired}$ $Check := if(A_{sProvided} > A_{sRequired}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Moment capacity of the section with the provided steel

$$M_{CapacityWall} := \phi_f \cdot A_{sProvided} \cdot f_y \cdot \frac{\left[d_e - \frac{1}{2} \cdot \left(\frac{A_{sProvided} \cdot f_y}{0.85 \cdot f_c \cdot b}\right)\right]}{ft}$$

$$M_{CapacityWall} = 153.09 \cdot \frac{kip \cdot ft}{ft}$$

Distance from the extreme compression fiber to the neutral axis

$$c := \frac{A_{sProvided} \cdot f_y}{0.85 \cdot f_c \cdot \beta_1 \cdot b} = 2.31 \cdot in$$

Check the validity of assumption, $f_s = f_y$

$$Check := if\left(\frac{c}{d_e} < 0.6, "OK", "Not\ OK"\right) = "OK"$$

Limits for Reinforcement

LRFD 5.6.3.3

The tensile reinforcement provided must be adequate to develop a factored flexural resistance at least equal to the lesser of the cracking moment or 1.33 times the factored moment from the applicable strength limit state load combinations.

Flexural cracking variability factor $\gamma_1 := 1.6$ For concrete structures that are not precast segmental

Ratio of specified minimum yield strength to ultimate tensile strength of the nonprestressed reinforcement $\gamma_3 := 0.67$ For ASTM615 grade 60 reinforcement

Section modulus

$$S_c := \frac{1}{6} \cdot b \cdot t_{wall}^2 = 2.89 \times 10^3 \cdot in^3$$

Cracking moment

$$M_{cr} := \frac{\gamma_3 \cdot \gamma_1 \cdot f_r \cdot S_c}{ft} = 107.25 \cdot \frac{kip \cdot ft}{ft}$$

1.33 times the factored moment demand

$$1.33 \cdot M_{DemandWall} = 174.29 \cdot \frac{kip \cdot ft}{ft}$$

The factored moment to satisfy the minimum reinforcement requirement

$$M_{req} := \min\left(1.33 M_{DemandWall}, M_{cr}\right) = 107.25 \cdot \frac{kip \cdot ft}{ft}$$

Check the adequacy of the section capacity

$$Check := if\left(M_{CapacityWall} > M_{req}, "OK", "Not\ OK"\right) = "OK"$$

Control of Cracking by Distribution of Reinforcement

LRFD 5.6.7

Limiting the width of expected cracks under service conditions extends the service life. The width of potential cracks can be minimized through proper placement of the reinforcement. The check for crack control requires that the actual stress in the reinforcement should not exceed the service limit state stress.

Spacing requirement for the mild steel reinforcement in the layer closest to the tension face

$$s \le \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

LRFD Eq. 5.6.7-1

Exposure factor for the Class 1 exposure condition $\gamma_e := 1.00$

Distance from extreme tension fiber to center of the closest flexural reinforcement	$d_c := Cover_{wall} = 3 \cdot in$
Ratio of flexural strain at the extreme tension face to the strain at the centroid of the reinforcement layer closest to the tension face	$\beta_s := 1 + \dfrac{d_c}{0.7\left(t_{wall} - d_c\right)} = 1.12$

The position of the cross-section's neutral axis is determined through an iterative process to calculate the actual stress in the reinforcement. This process starts with an assumed position of the neutral axis as shown below.

Assumed distance from the extreme compression fiber to the neutral axis	$x := 6 \cdot in$
Given	$\dfrac{1}{2} \cdot b \cdot x^2 = \dfrac{E_s}{E_c} \cdot A_{sProvided} \cdot \left(d_e - x\right)$
Position of the neutral axis	$x_{na} := Find(x) = 6.197 \cdot in$
Tensile force in the reinforcing steel due to service limit state moment	$T_s := \dfrac{M_{uWallLC4SerI}}{d_e - \dfrac{x_{na}}{3}} \cdot ft = 32.1 \cdot kip$
Stress in the reinforcing steel due to service limit state moment	$f_{ss1} := \dfrac{T_s}{A_{sProvided}} = 32.12 \cdot ksi$
f_{ss} (not to exceed $0.6f_y$)	$f_{ss} := \min\left(f_{ss1}, 0.6f_y\right) = 32.12 \cdot ksi$
Required reinforcement spacing	$s_{barRequred} := \dfrac{700 \cdot \gamma_e \cdot \dfrac{kip}{in}}{\beta_s \cdot f_{ss}} - 2 \cdot d_c = 13.42 \cdot in$
Check if the spacing provided < the required spacing	$Check := if\left(s_{bar} < s_{barRequred}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Shrinkage and Temperature Reinforcement Requirement

LRFD 5.10.6

The following calculations check the adequacy of the flexural reinforcing steel to control shrinkage and temperature stresses in the wall:

Minimum area of shrinkage and temperature reinforcement	$A_{shrink.temp} := \min\left[\left[\begin{array}{c} \left(0.60 \dfrac{in^2}{ft}\right) \\ \max\left[\left[\begin{array}{c} \left(0.11 \dfrac{in^2}{ft}\right) \\ \left[\dfrac{1.3 \cdot h_{wall} \cdot t_{wall} \cdot \dfrac{kip}{in \cdot ft}}{2\left(h_{wall} + t_{wall}\right) \cdot f_y}\right] \end{array}\right]\right] \end{array}\right]\right] \cdot ft = 0.35 \cdot in^2$
Check if the provided area of steel > the required area of shrinkage and temperature steel	$Check := if\left(A_{sProvided} > A_{shrink.temp}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Design for Shear

According to the loads in the summary tables, Load Case IV under Strength I limit state is the governing load case for the shear design.

The maximum factored shear force at the base of the abutment wall $V_{uWallLC4StrI} = 15.7 \cdot \frac{kip}{ft}$

Effective width of the section $b_v := b = 12 \cdot in$

Depth of the equivalent rectangular stress block $a := \frac{A_{sProvided} \cdot f_y}{0.85 \cdot f_c \cdot b} = 1.96 \cdot in$

Effective shear depth $d_v := \max\left(d_e - \frac{a}{2}, 0.9 \cdot d_e, 0.72 \cdot t_{wall}\right) = 34.02 \cdot in$ **LRFD 5.7.2.8**

Note: Since there is no transverse reinforcement in the wall and the overall depth of the wall is greater than 16 in., the simplified procedure in LRFD 5.7.3.4.1 cannot be used. The general procedure outlined in LRFD 5.7.3.4.2 is used for the calculation of abutment wall shear capacity.

The factored N_u, V_u, and M_u are calculated at the critical section for shear, which is located at a distance d_v from the base of the abutment wall.

Factored axial force at the critical section (use negative if compression)

$$N_{uWallShear} := -\left[1.25 \cdot \left(DC_{Sup} + DC_{backwall} + DC_{wall} - d_v \cdot t_{wall} \cdot W_c\right) + 1.5 DW_{Sup}\right] = -18.33 \cdot \frac{kip}{ft}$$

Lateral earth load at the critical section

$$P_{EHWallShear} := \frac{1}{2} \cdot \left[k_a \cdot \gamma_s \cdot \left(h_{backwall} + h_{wall} - d_v\right)\right] \cdot \left(h_{backwall} + h_{wall} - d_v\right)$$

$$P_{EHWallShear} = 6.47 \cdot \frac{kip}{ft}$$

Load at the critical section due to live load surcharge

$$P_{LSWallShear} := k_a \cdot \gamma_s \cdot h_{eq} \cdot \left(h_{backwall} + h_{wall} - d_v\right) = 1.36 \cdot \frac{kip}{ft}$$

Factored shear force (demand) at the critical section

$$V_{uWallShear} := 1.5 \cdot P_{EHWallShear} + 1.75 \cdot P_{LSWallShear} + 0.5 TU$$

$$V_{uWallShear} = 12.23 \cdot \frac{kip}{ft}$$

Factored moment at the critical section

$$M_{uWallShear} := 0.9 \cdot DC_{backwall} \cdot \frac{\left(t_{backwall} - t_{wall}\right)}{2} + \left(1.25 \cdot DC_{Sup} + 1.5 \cdot DW_{Sup}\right) \cdot \left(l_{brtowall} - \frac{t_{wall}}{2}\right) \ldots$$
$$+ 1.5 \cdot P_{EHWallShear} \cdot \frac{\left(h_{backwall} + h_{wall} - d_v\right)}{3} \ldots$$
$$+ 1.75 \cdot P_{LSWallShear} \cdot \frac{\left(h_{backwall} + h_{wall} - d_v\right)}{2} + 0.5 \cdot TU \cdot \left(h_{wall} - d_v\right)$$

$$M_{uWallShear} = 91.55 \cdot \frac{kip \cdot ft}{ft}$$

Check M_u since it cannot be taken less than $V_u d_v$

$$M_{uWallShear} := \max\left(M_{uWallShear}, V_{uWallShear} \cdot d_v\right) = 91.55 \cdot \frac{kip \cdot ft}{ft}$$

Net longitudinal tensile strain in the section at the centroid of the tension reinforcement

$$\varepsilon_s := \frac{\left(\frac{M_{uWallShear}}{d_v} + 0.5 \cdot N_{uWallShear} + V_{uWallShear}\right)}{E_s \cdot \frac{A_{sProvided}}{ft}} = 1.22 \times 10^{-3}$$

LRFD Eq. 5.7.3.4.2-4

Crack spacing parameter $s_x := d_v = 2.83\ ft$

Maximum aggregate size (in.) $a_g := 1.5$

MDOT Standard Specifications for Construction Table 902-1

Crack spacing parameter as influenced by the maximum aggregate size

$$s_{xe} := \min\left[\left[\max\left[\left[\begin{pmatrix} (80in) \\ (12in) \\ \left(s_x \cdot \frac{1.38}{a_g + 0.63}\right)\end{pmatrix}\right]\right]\right]\right] = 22.04 \cdot in$$

LRFD Eq. 5.7.3.4.2-7

Factor indicating the ability of diagonally cracked concrete to transmit tension and shear

$$\beta := \frac{4.8}{(1 + 750 \cdot \varepsilon_s)} \cdot \frac{51}{\left(39 + \frac{s_{xe}}{in}\right)} = 2.09$$

LRFD Eq. 5.7.3.4.2-2

Nominal shear resistance of concrete, V_n, is calculated as follows:

$$V_{c1} := 0.0316 \cdot \beta \cdot \sqrt{f_c \cdot ksi} \cdot b \cdot d_e = 48.2 \cdot kip$$

LRFD Eq. 5.7.3.3-3

$$V_{c2} := 0.25 f_c \cdot b \cdot d_e = 315 \cdot kip$$

LRFD Eq. 5.7.3.3-2

$$V_n := \min(V_{c1}, V_{c2}) = 48.16 \cdot kip$$

Resistance factor for shear $\phi_v := 0.9$ **LRFD 5.5.4.2**

Factored shear resistance (capacity) $V_r := \phi_v \cdot V_n = 43.34 \cdot kip$

Check if the shear demand is greater than the demand

$$Check := if\left(\frac{V_r}{ft} > V_{uWallShear}, "OK", "Not\ OK"\right) = "OK"$$

Development Length of Reinforcement

The flexural reinforcing steel must be developed on each side of the critical section for its full development length.

LRFD 5.10.8.1.2, 5.10.8.2.1

Basic development length

$$l_{db} := 2.4 \cdot d_{bar} \cdot \frac{f_y}{\sqrt{f_c \cdot ksi}} = 7.82\ ft$$

LRFD Eq. 5.10.8.2.1a-2

Reinforcement location factor $\lambda_{rl} := 1$ No more than 12 in. concrete below

Coating factor $\lambda_{cf} := 1.5$ Epoxy coated bars with less than $3d_b$ cover

Distance from center of the bar to the nearest concrete surface $c_b := Cover_{wall} = 3 \cdot in$

Reinforcement confinement factor $\lambda_{rc} := \frac{d_{bar}}{c_b} = 0.38$

Excess reinforcement factor $\lambda_{er} := \frac{A_{sRequired}}{A_{sProvided}} = 0.85$

Factor for normal weight concrete $\lambda := 1$

Required development length $l_d := l_{db} \cdot \frac{(\lambda_{rl} \cdot \lambda_{cf} \cdot \lambda_{rc} \cdot \lambda_{er})}{\lambda} = 3.76\ ft$ **LRFD Eq. 5.10.8.2.1a-1**

Since the footing thickness is 3 ft, an adequate space is not available for straight bars. The common practice is to use hooked bars which are set on the bottom reinforcing steel layer.

Shrinkage and Temperature Reinforcement

The following calculations check the required amount of reinforcing steel in the secondary direction to control shrinkage and temperature stresses in the abutment wall.

The reinforcement at the front face of the abutment wall and the horizontal reinforcement at the interior should satisfy the shrinkage and temperature reinforcement requirements. **LRFD 5.10.6**

The spacing of reinforcement shall not exceed 12 in. when the wall thickness is greater than 18 in. **LRFD 5.10.6**

Note: MDOT practice is to use 18 in. as the maximum spacing. **BDG 5.16.01**

As a trial, select No. 6 bars. $bar := 6$

Nominal diameter of a reinforcing steel bar $d_{bST} := Dia(bar) = 0.75 \cdot in$

Cross-section area of the bar $A_{barST} := Area(bar) = 0.44 \cdot in^2$

Spacing of bars $s_{barST} := 12 \cdot in$

Reinforcing steel area provided in the section $A_{sProvidedST} := \frac{A_{barST} \cdot 12in}{s_{barST}} = 0.44 \cdot in^2$

The required minimum shrinkage and temperature reinforcement area at the abutment wall was previously calculated during the design of flexural reinforcement.

Required shrinkage and temperature steel area $A_{shrink.temp} = 0.35 \cdot in^2$

Check if the provided steel area > the required area of shrinkage and temperature steel $Check := if(A_{sProvidedST} > A_{shrink.temp}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

The abutment wall design presented in this step provides the following details:

- No. 9 bars at 12.0 in. spacing ($A_s = 1.0$ in.^{2}/ft) as the back face flexural reinforcement.
- No. 6 bars at 12.0 in. spacing ($A_s = 0.44$ in.^{2}/ft) as the front face vertical shrinkage and temperature reinforcement.
- No. 6 bars at 12.0 in. spacing ($A_s = 0.44$ in.^{2}/ft) as the front and back face horizontal shrinkage and temperature reinforcement.

Step 2.9 Structural Design of the Footing

Description

This step presents the structural design of the abutment footing.

Page Contents

Forces and Moments at the Base of the Abutment Footing

Step 2.5 presents the load effects at the base of the footing under different load cases and limit states. A summary is presented in the following tables:

Factored vertical force, F_{VFt} (kip/ft)

	Strength I	Service I
LC I	56.79	43.32
LC III	74.75	55.33
LC IV	69.08	52.09

Factored shear force parallel to the transverse axis of the footing, V_{uFt} (kip/ft)

	Strength I	Service I
LC I	16.59	11.06
LC III	16.59	11.06
LC IV	19.85	13.12

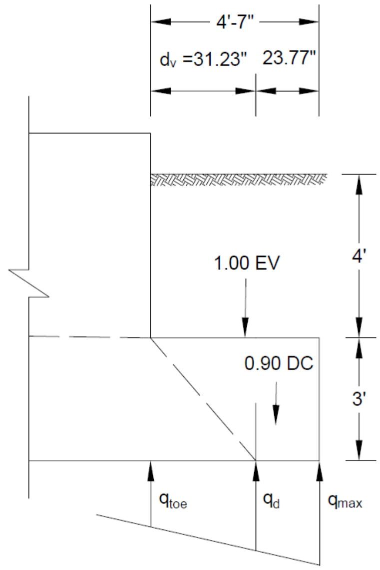
Factored moment about the longitudinal axis of the footing, M_{uFt} (kip·ft/ft)

	Strength I	Service I
LC I	87.92	32.22
LC III	143.27	69.22
LC IV	140.34	71.62

Note: In this example, the length of the footing and the abutment wall are 65.75 ft and 63.75 ft, respectively. Since the cantilevered length of the footing in the longitudinal direction is limited to 1 ft on each side, the shear and moment acting on the footing in the longitudinal direction are small and do not require flexural and shear designs.

Toe Design

The necessary dimensions, loads, and the bearing pressure distribution are shown in the following figure:



For structural design of an eccentrically loaded foundation, a triangular or trapezoidal bearing pressure distribution is used. **LRFD 10.6.5**

According to the loads in the summary tables, Load Case III under Strength I limit state is identified as the governing load case for the design of flexure and shear at the toe.

$$F_{VFtLC3StrI} = 74.75 \cdot \frac{kip}{ft} \qquad M_{uFtLC3StrI} = 143.27 \cdot \frac{kip \cdot ft}{ft}$$

Eccentricity in the footing width direction

$$e_B := \frac{M_{uFtLC3StrI}}{F_{VFtLC3StrI}} = 1.92 \cdot ft$$

Maximum and minimum bearing pressure

$$q_{max} := \frac{F_{VFtLC3StrI}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}}\right) = 7.37 \cdot ksf$$

$$q_{min} := \frac{F_{VFtLC3StrI}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}}\right) = 1.42 \cdot ksf$$

The critical section for flexural design is at the front face of the wall. **LRFD 5.12.8.4**

Bearing pressure at the critical section

$$q_{toe} := q_{min} + \frac{\left(q_{max} - q_{min}\right)}{B_{footing}} \cdot \left(B_{footing} - l_{toe}\right) = 5.77 \cdot ksf$$

A simplified analysis method is used in this example to determine the maximum moments at the front face of the wall by selecting load factors to produce the maximum bearing pressure and minimum resisting loads. This method is conservative and eliminates the need for using multiple combinations.

As shown below, minimum load factors are used for the resisting forces such as the overburden and footing self-weight to calculate the maximum moment at the front face of the wall.

The moment demand at the critical section

$$M_{rDemand} := q_{toe}\cdot\frac{l_{toe}^{2}}{2} + (q_{max} - q_{toe})\cdot\frac{l_{toe}^{2}}{3} - 0.9\cdot W_c\cdot t_{footing}\cdot\frac{l_{toe}^{2}}{2} - 1.0\gamma_s\cdot(h_{toeDepth} - t_{footing})\cdot\frac{l_{toe}^{2}}{2}$$

$$M_{rDemand} = 62.51\cdot\frac{kip\cdot ft}{ft}$$

Flexural Resistance **LRFD 5.6.3.2**

The design procedure consists of calculating the reinforcing steel area required to satisfy the moment demand and checking the selected steel area against the requirements and limitations for developing an adequate moment capacity, controlling crack width, and managing shrinkage and temperature stresses.

As a trial, select No. 8 bars. $bar := 8$

Nominal diameter of a reinforcing steel bar $d_{bar} := Dia(bar) = 1\cdot in$

Cross-section area of a bar on the flexural tension side $A_{bar} := Area(bar) = 0.79\cdot in^2$

The spacing shall not exceed 12 in. when the footing thickness is greater than 18 in. **LRFD 5.10.6**

Footing thickness $t_{footing} = 3\,ft$

Selected spacing of reinforcing steel bars $s_{bar} := 12\cdot in$

Area of tension steel provided in a 1-ft wide strip $A_{sProvided} := \frac{A_{bar}\cdot 12in}{s_{bar}} = 0.79\cdot in^2$

Effective depth $d_e := t_{footing} - Cover_{ft} = 32\cdot in$

Resistance factor for flexure $\phi_f := 0.9$ **LRFD 5.5.4.2**

A 1-ft wide strip is selected for the design.

Width of the compression face of the section $b := 12in$

Stress block factor $\beta_1 = 0.85$

Solve the following equation of A_s to calculate the required area of steel to satisfy the moment demand. Use an assumed initial A_s value to solve the equation.

Initial assumption $A_s := 1in^2$

Given $$M_{rDemand}\cdot ft = \phi_f\cdot A_s\cdot f_y\cdot\left[d_e - \frac{1}{2}\cdot\left(\frac{A_s\cdot f_y}{0.85\cdot f_c\cdot b}\right)\right]$$

Required area of steel $A_{sRequired} := Find(A_s) = 0.44\cdot in^2$

Check if $A_{sProvided} > A_{sRequired}$ $Check := if(A_{sProvided} > A_{sRequired}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Moment capacity of the section with the provided steel

$$M_{Provided} := \phi_f \cdot A_{sProvided} \cdot f_y \cdot \frac{\left[d_e - \frac{1}{2}\cdot\left(\frac{A_{sProvided}\cdot f_y}{0.85\cdot f_c \cdot b}\right)\right]}{ft}$$

$$M_{Provided} = 111.01 \cdot \frac{kip \cdot ft}{ft}$$

Distance from the extreme compression fiber to the neutral axis

$$c := \frac{A_{sProvided}\cdot f_y}{0.85\cdot f_c\cdot \beta_1 \cdot b} = 1.82 \cdot in$$

Check the validity of assumption, $f_s = f_y$

$$Check\_f_s := if\left(\frac{c}{d_e} < 0.6, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Limits for Reinforcement

LRFD 5.6.3.3

The tensile reinforcement provided must be adequate to develop a factored flexural resistance at least equal to the lesser of the cracking moment or 1.33 times the factored moment from the applicable strength limit state load combinations.

Flexural cracking variability factor

$\gamma_1 := 1.6$ For concrete structures that are not precast segmental

Ratio of specified minimum yield strength to ultimate tensile strength of the nonprestressed reinforcement

$\gamma_3 := 0.67$ For ASTM615 grade 60 reinforcement

Section modulus

$$S_c := \frac{1}{6}\cdot b \cdot t_{footing}^{\ 2} = 2.59 \times 10^3 \cdot in^3$$

Cracking moment

$$M_{cr} := \frac{\gamma_3 \cdot \gamma_1 \cdot f_r \cdot S_c}{ft} = 96.25 \cdot \frac{kip \cdot ft}{ft}$$

1.33 times the factored moment demand

$$1.33 \cdot M_{rDemand} = 83.14 \cdot \frac{kip \cdot ft}{ft}$$

The factored moment to satisfy the minimum reinforcement requirement

$$M_{req} := \min\left(1.33 M_{rDemand}, M_{cr}\right) = 83.14 \cdot \frac{kip \cdot ft}{ft}$$

Check the adequacy of section capacity

$$Check := if\left(M_{Provided} > M_{req}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Control of Cracking by Distribution of Reinforcement

LRFD 5.6.7

Limiting the width of expected cracks under service conditions extends the service life. The width of potential cracks can be minimized through proper placement of the reinforcement. The check for crack control requires that the actual stress in the reinforcement should not exceed the service limit state stress.

The spacing requirement for the mild steel reinforcement in the layer closest to the tension face

$$s \le \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

LRFD Eq. 5.6.7-1

Exposure factor for the Class 1 exposure condition

$\gamma_e := 1.00$

Distance from extreme tension fiber to the center of the closest flexural reinforcement

$$d_c := Cover_{ft} = 4 \cdot in$$

Ratio of flexural strain at the extreme tension face to the strain at the centroid of the reinforcement layer closest to the tension face

$$\beta_s := 1 + \frac{d_c}{0.7\left(t_{footing} - d_c\right)} = 1.18$$

The calculation of tensile stress in nonprestressed reinforcement at the service limit state, f_{ss}, requires establishing the neutral axis location and the moment demand at the critical section.

The position of the cross-section's neutral axis is determined through an iterative process to calculate the actual stress in the reinforcement. This process starts with an assumed position of the neutral axis as shown below.

Assumed distance from the extreme compression fiber to the neutral axis

$$x := 5 \cdot in$$

Given

$$\frac{1}{2} \cdot b \cdot x^2 = \frac{E_s}{E_c} \cdot A_{sProvided} \cdot \left(d_e - x\right)$$

Position of the neutral axis

$$x_{na} := Find(x) = 5.303 \cdot in$$

Vertical force and moment at the base of the footing from Load Case III under Service I limit state are:

$$F_{VFtLC3SerI} = 55.33 \cdot \frac{kip}{ft} \qquad M_{uFtLC3SerI} = 69.22 \cdot \frac{kip \cdot ft}{ft}$$

Eccentricity in the footing width direction under Service I limit state

$$e_{BSerI} := \frac{M_{uFtLC3SerI}}{F_{VFtLC3SerI}} = 1.25 \cdot ft$$

Maximum and minimum bearing pressure under Service I limit state

$$q_{maxSerI} := \frac{F_{VFtLC3SerI}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_{BSerI}}{B_{footing}}\right) = 4.69 \cdot ksf$$

$$q_{minSerI} := \frac{F_{VFtLC3SerI}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_{BSerI}}{B_{footing}}\right) = 1.82 \cdot ksf$$

Soil pressure at the critical section under Service I limit state

$$q_{toeSerI} := q_{minSerI} + \frac{\left(q_{maxSerI} - q_{minSerI}\right)}{B_{footing}} \cdot \left(B_{footing} - l_{toe}\right)$$

$$q_{toeSerI} = 3.92 \cdot ksf$$

The moment at the critical section under Service I limit state:

$$M_{rSerI} := q_{toeSerI} \cdot \frac{l_{toe}^2}{2} + \left(q_{maxSerI} - q_{toeSerI}\right) \cdot \frac{l_{toe}^2}{3} - W_c \cdot t_{footing} \cdot \frac{l_{toe}^2}{2} - \gamma_s \cdot \left(h_{toeDepth} - t_{footing}\right) \cdot \frac{l_{toe}^2}{2}$$

$$M_{rSerI} = 36.8 \cdot \frac{kip \cdot ft}{ft}$$

Tensile force in the reinforcing steel due to the service limit state moment

$$T_s := \frac{M_{rSerI}}{d_e - \frac{x_{na}}{3}} \cdot ft = 14.6 \cdot kip$$

Stress in the reinforcing steel due to the service limit state moment

$$f_{ss1} := \frac{T_s}{A_{sProvided}} = 18.49 \cdot ksi$$

f_{ss} (not to exceed $0.6f_y$)

$$f_{ss} := \min\left(f_{ss1}, 0.6f_y\right) = 18.49 \cdot ksi$$

Required reinforcement spacing

$$s_{barRequired} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{ss}} - 2 \cdot d_c = 24.13 \cdot in$$

Check if the spacing provided < the required spacing

$$Check := if\left(s_{bar} < s_{barRequired}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Shrinkage and Temperature Reinforcement Requirement

LRFD 5.10.6

The following calculations check the adequacy of the flexural reinforcing steel to control shrinkage and temperature stresses in the footing:

Minimum area of shrinkage and temperature reinforcement

$$A_{shrink.temp} := \min\left[\left[\begin{matrix}\left(0.60 \frac{in^2}{ft}\right) \\ \max\left[\left[\begin{matrix}\left(0.11 \frac{in^2}{ft}\right) \\ \left[\frac{1.3 \cdot B_{footing} \cdot t_{footing} \cdot \frac{kip}{in \cdot ft}}{2\left(B_{footing} + t_{footing}\right) \cdot f_y}\right]\end{matrix}\right]\right]\end{matrix}\right]\right] \cdot ft = 0.33 \cdot in^2$$

Check if the provided area of steel > the required area of shrinkage and temperature steel

$$Check := if\left(A_{sProvided} > A_{shrink.temp}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Design for Shear

Effective width of the section

$$b = 12 \cdot in$$

Depth of the equivalent rectangular stress block

$$a := \frac{A_{sProvided} \cdot f_y}{0.85 \cdot f_c \cdot b} = 1.55 \cdot in$$

Effective shear depth

$$d_v := \max\left(d_e - \frac{a}{2}, 0.9 \cdot d_e, 0.72 \cdot t_{footing}\right) = 31.23 \cdot in$$

LRFD 5.7.2.8

The critical section for shear at the toe is located at a distance d_v from the front face of the wall.

Distance from the toe to the critical section

$$l_{shear} := l_{toe} - d_v = 1.98\ ft$$

Bearing pressure at the critical section

$$q_d := q_{min} + \frac{\left(q_{max} - q_{min}\right)}{B_{footing}} \cdot \left(B_{footing} - l_{shear}\right) = 6.68 \cdot ksf$$

Minimum load factors are used for the resisting forces such as the overburden and footing self-weight to calculate the maximum shear at the critical section.

Factored shear force (demand) at the critical section

$$V_{uFtToe} := \frac{\left(q_{max} + q_d\right)}{2} \cdot l_{shear} - 0.9 \cdot W_c \cdot t_{footing} \cdot l_{shear} - 1.0 \cdot \gamma_s \cdot \left(h_{toeDepth} - t_{footing}\right) \cdot l_{shear} = 12.16 \cdot \frac{kip}{ft}$$

The simplified procedure for nonprestressed sections can be used for the design of shear in concrete footings when the distance from the point of zero shear to the face of the wall is less than $3d_v$. **LRFD 5.7.3.4.1**

Check if the distance l_{toe} is less than $3d_v$ $\quad Check := if\left(l_{toe} < 3 \cdot d_v, "Yes", "No"\right) = "Yes"$

Therefore, the simplified procedure is used.

Factor indicating the ability of diagonally cracked concrete to transmit tension and shear $\quad \beta := 2$

Nominal shear resistance of concrete, V_n, is calculated as follows:

$$V_{c1} := 0.0316 \cdot \beta \cdot \sqrt{f_c \cdot ksi} \cdot b \cdot d_e = 42 \cdot kip$$ **LRFD Eq. 5.7.3.3-3**

$$V_{c2} := 0.25 f_c \cdot b \cdot d_e = 288 \cdot kip$$ **LRFD Eq. 5.7.3.3-2**

$$V_n := \min\left(V_{c1}, V_{c2}\right) = 42.03 \cdot kip$$

Resistance factor for shear $\quad \phi_v := 0.9$ **LRFD 5.5.4.2**

Factored shear resistance (demand) $\quad V_r := \phi_v \cdot V_n = 37.83 \cdot kip$

Check if the shear capacity is greater than the demand $\quad Check := if\left(\frac{V_r}{ft} > V_{uFtToe}, "OK", "Not OK"\right) = "OK"$

Development Length of Reinforcement

The flexural reinforcing steel must be developed on each side of the critical section for its full development length. **LRFD 5.10.8.1.2**

Available length for rebar development $\quad l_{d.available} := l_{toe} - Cover_{ft} = 4.25\ ft$

Basic development length $\quad l_{db} := 2.4 \cdot d_{bar} \cdot \frac{f_y}{\sqrt{f_c \cdot ksi}} = 6.93\ ft$ **LRFD Eq. 5.10.8.2.1a-2**

Reinforcement location factor $\quad \lambda_{rl} := 1 \quad$ No more than 12 in. concrete below

Coating factor $\quad \lambda_{cf} := 1.5 \quad$ Epoxy coating bars with less than $3d_b$ cover

Reinforcement confinement factor $\quad \lambda_{rc} := 0.4 \quad$ For $c_b > 2.5$ in and No. 8 bars or smaller

Excess reinforcement factor $\quad \lambda_{er} := \frac{A_{sRequired}}{A_{sProvided}} = 0.56$ **LRFD Eq. 5.10.8.2.1c-4**

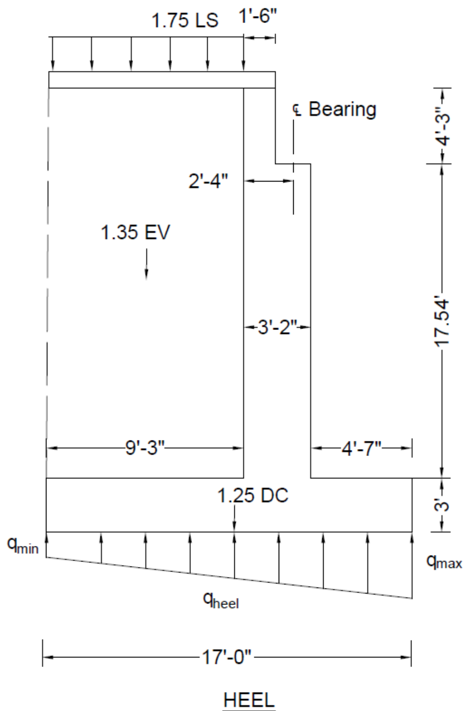
Factor for normal weight concrete $\quad \lambda := 1$

Required development length $\quad l_{d.required} := l_{db} \cdot \frac{\left(\lambda_{rl} \cdot \lambda_{cf} \cdot \lambda_{rc} \cdot \lambda_{er}\right)}{\lambda} = 2.32\ ft$ **LRFD Eq. 5.10.8.2.1a-1**

Check if $l_{d.available} > l_{d.required}$ $\quad Check := if\left(l_{d.available} > l_{d.required}, "OK", "Not OK"\right) = "OK"$

Heel Design

The necessary dimensions, loads, and the bearing pressure distribution are shown in the following figure:



The self-weight of the footing, the weight of soil, live load surcharge and the bearing pressure act on the heel. The critical load combination for the design selects the load factors to produce the minimum vertical loads and maximum eccentricities resulting in the minimum bearing pressure.

The critical location for the design of flexure is located at the back face of the wall. **LRFD 5.12.8.4**

In the general case of a cantilever abutment wall, where the downward load on the heel is larger than the upward reaction of the soil under the heel, the top of the heel is in tension. Therefore, the critical section for shear is taken at the back face of the abutment wall. **LRFD C5.12.8.6.1**

Load cases I, III, and IV under Strength I limit state are used to calculate the maximum moment and shear at the critical sections.

<u>Load Case I</u>

Minimum vertical force $F_{VFtLC1StrIMin} = 41.63 \cdot \frac{kip}{ft}$ **Step 2.6, sliding resistance check**

Factored moment about the longitudinal axis of the footing $M_{uFtLC1StrI} = 87.92 \cdot \frac{kip \cdot ft}{ft}$ **Step 2.6, summary table**

Eccentricity in the footing width direction $e_B := \frac{M_{uFtLC1StrI}}{F_{VFtLC1StrIMin}} = 2.11 \cdot ft$

Maximum and minimum bearing pressure

$$q_{max} := \frac{F_{VFtLC1StrIMin}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}}\right) = 4.27 \cdot ksf$$

$$q_{min} := \frac{F_{VFtLC1StrIMin}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}}\right) = 0.62 \cdot ksf$$

Bearing pressure at the critical section

$$q_{heelLC1StrI} := q_{min} + (q_{max} - q_{min}) \frac{l_{heel}}{B_{footing}} = 2.61 \cdot ksf$$

Factored moment at the critical section

$$M_{rLC1StrI} := 1.25 \cdot W_c \cdot t_{footing} \cdot \frac{l_{heel}^2}{2} + 1.35 EV_{earthBk} \cdot \frac{l_{heel}}{2} - q_{min} \cdot l_{heel} \cdot \frac{l_{heel}}{2} - \frac{1}{6} (q_{heelLC1StrI} - q_{min}) l_{heel}^2$$

$$M_{rLC1StrI} = 120.08 \cdot \frac{kip \cdot ft}{ft}$$

Factored shear force at the critical section

$$V_{uHeelLC1StrI} := 1.25 \cdot W_c \cdot t_{footing} \cdot l_{heel} + 1.35 EV_{earthBk} - q_{min} \cdot l_{heel} - \frac{1}{2} \cdot (q_{heelLC1StrI} - q_{min}) \cdot l_{heel}$$

$$V_{uHeelLC1StrI} = 22.9 \cdot \frac{kip}{ft}$$

Load Case III

There are two cases that need to be considered: without and with the live load.

Without the live load

Minimum vertical force

$$F_{VFtLC3StrIMin\_noLL} = 46.72 \cdot \frac{kip}{ft}$$

Step 2.6, sliding resistance check

Factored moment about the longitudinal axis of the footing

$$M_{uFtLC3StrI\_noLL} = 113.82 \cdot \frac{kip \cdot ft}{ft}$$

Step 2.6, eccentric load limitation check

Eccentricity in the footing width direction

$$e_B := \frac{M_{uFtLC3StrI\_noLL}}{F_{VFtLC3StrIMin\_noLL}} = 2.44 \cdot ft$$

Maximum and minimum bearing pressure

$$q_{max} := \frac{F_{VFtLC3StrIMin\_noLL}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}}\right) = 5.11 \cdot ksf$$

$$q_{min} := \frac{F_{VFtLC3StrIMin\_noLL}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}}\right) = 0.39 \cdot ksf$$

Bearing pressure at the critical section

$$q_{heelLC3StrI} := q_{min} + (q_{max} - q_{min}) \frac{l_{heel}}{B_{footing}} = 2.96 \cdot ksf$$

Factored moment at the critical section

$$M_{rLC3StrI\_noLL} := 1.25 \cdot W_c \cdot t_{footing} \cdot \frac{l_{heel}^2}{2} + 1.35 EV_{earthBk} \cdot \frac{l_{heel}}{2} - q_{min} \cdot l_{heel} \cdot \frac{l_{heel}}{2} - \frac{1}{6} \left(q_{heelLC3StrI} - q_{min} \right) l_{heel}^2$$

$$M_{rLC3StrI\_noLL} = 121.93 \cdot \frac{kip \cdot ft}{ft}$$

Factored shear force at the critical section

$$V_{uHeelLC3StrI\_noLL} := 1.25 \cdot W_c \cdot t_{footing} \cdot l_{heel} + 1.35 EV_{earthBk} - q_{min} \cdot l_{heel} - \frac{1}{2} \cdot \left(q_{heelLC3StrI} - q_{min} \right) \cdot l_{heel}$$

$$V_{uHeelLC3StrI\_noLL} = 22.4 \cdot \frac{kip}{ft}$$

With the live load

Minimum vertical force

$$F_{VFtLC3StrIMin} = 56.27 \cdot \frac{kip}{ft}$$

Step 2.6, sliding resistance check

Factored moment about the longitudinal axis of the footing

$$M_{uFtLC3StrI} = 143.27 \cdot \frac{kip \cdot ft}{ft}$$

Step 2.6, summary table

Eccentricity in the footing width direction

$$e_B := \frac{M_{uFtLC3StrI}}{F_{VFtLC3StrIMin}} = 2.55 \cdot ft$$

Maximum and minimum bearing pressure

$$q_{max} := \frac{F_{VFtLC3StrIMin}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}} \right) = 6.28 \cdot ksf$$

$$q_{min} := \frac{F_{VFtLC3StrIMin}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}} \right) = 0.34 \cdot ksf$$

Bearing pressure at the critical section

$$q_{heelLC3StrI} := q_{min} + \left(q_{max} - q_{min} \right) \frac{l_{heel}}{B_{footing}} = 3.57 \cdot ksf$$

Factored moment at the critical section

$$M_{rLC3StrI} := 1.25 \cdot W_c \cdot t_{footing} \cdot \frac{l_{heel}^2}{2} + 1.35 EV_{earthBk} \cdot \frac{l_{heel}}{2} - q_{min} \cdot l_{heel} \cdot \frac{l_{heel}}{2} - \frac{1}{6} \left(q_{heelLC3StrI} - q_{min} \right) l_{heel}^2$$

$$M_{rLC3StrI} = 114.56 \cdot \frac{kip \cdot ft}{ft}$$

Factored shear force at the critical section

$$V_{uHeelLC3StrI} := 1.25 \cdot W_c \cdot t_{footing} \cdot l_{heel} + 1.35 EV_{earthBk} - q_{min} \cdot l_{heel} - \frac{1}{2} \cdot \left(q_{heelLC3StrI} - q_{min} \right) \cdot l_{heel}$$

$$V_{uHeelLC3StrI} = 19.78 \cdot \frac{kip}{ft}$$

Load Case IV

There are two cases that need to be considered: without and with the live load surcharge.

Without the live load surcharge

Minimum vertical force

$$F_{VFtLC4StrIMin\_noLS} = 46.72 \cdot \frac{kip}{ft}$$

Step 2.6, sliding resistance check

Factored moment about the longitudinal axis of the footing

$$M_{uFtLC4StrI\_noLS} = 116.67 \cdot \frac{kip \cdot ft}{ft}$$

Step 2.6, eccentric load limitation check

Eccentricity in the footing width direction

$$e_B := \frac{M_{uFtLC4StrI\_noLS}}{F_{VFtLC4StrIMin\_noLS}} = 2.5 \cdot ft$$

Maximum and minimum bearing pressure

$$q_{max} := \frac{F_{VFtLC4StrIMin\_noLS}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}}\right) = 5.17 \cdot ksf$$

$$q_{min} := \frac{F_{VFtLC4StrIMin\_noLS}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}}\right) = 0.33 \cdot ksf$$

Bearing pressure at the critical section

$$q_{heelLC4StrI} := q_{min} + \left(q_{max} - q_{min}\right)\frac{l_{heel}}{B_{footing}} = 2.96 \cdot ksf$$

Factored moment at the critical section

$$M_{rLC4StrI\_noLS} := 1.25 \cdot W_c \cdot t_{footing} \cdot \frac{{l_{heel}}^2}{2} + 1.35EV_{earthBk} \cdot \frac{l_{heel}}{2} - q_{min} \cdot l_{heel} \cdot \frac{l_{heel}}{2} - \frac{1}{6}\left(q_{heelLC4StrI} - q_{min}\right) {l_{heel}}^2$$

$$M_{rLC4StrI\_noLS} = 123.54 \cdot \frac{kip \cdot ft}{ft}$$

Factored shear force at the critical section

$$V_{uHeelLC4StrI\_noLS} := 1.25 \cdot W_c \cdot t_{footing} \cdot l_{heel} + 1.35EV_{earthBk} - q_{min} \cdot l_{heel} - \frac{1}{2} \cdot \left(q_{heelLC4StrI} - q_{min}\right) \cdot l_{heel}$$

$$V_{uHeelLC4StrI\_noLS} = 22.65 \cdot \frac{kip}{ft}$$

With the live load surcharge

Minimum vertical force

$$F_{VFtLC4StrIMin} = 50.61 \cdot \frac{kip}{ft}$$

Step 2.6, sliding resistance check

Factored moment about the longitudinal axis of the footing

$$M_{uFtLC4StrI} = 140.34 \cdot \frac{kip \cdot ft}{ft}$$

Step 2.6, summary table

Eccentricity in the footing width direction

$$e_B := \frac{M_{uFtLC4StrI}}{F_{VFtLC4StrIMin}} = 2.77 \cdot ft$$

Maximum and minimum bearing pressure

$$q_{max} := \frac{F_{VFtLC4StrIMin}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}}\right) = 5.89 \cdot ksf$$

$$q_{min} := \frac{F_{VFtLC4StrIMin}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}}\right) = 0.06 \cdot ksf$$

Bearing pressure at the critical section

$$q_{heelLC4StrI} := q_{min} + \left(q_{max} - q_{min}\right) \frac{l_{heel}}{B_{footing}} = 3.23 \cdot ksf$$

Factored moment at the critical section

$$M_{rLC4StrI} := 1.25 \cdot W_c \cdot t_{footing} \cdot \frac{l_{heel}^2}{2} + 1.35 EV_{earthBk} \cdot \frac{l_{heel}}{2} - q_{min} \cdot l_{heel} \cdot \frac{l_{heel}}{2} - \frac{1}{6}\left(q_{heelLC4StrI} - q_{min}\right) l_{heel}^2$$

$$M_{rLC4StrI} = 127.15 \cdot \frac{kip \cdot ft}{ft}$$

Factored shear force at the critical section

$$V_{uHeelLC4StrI} := 1.25 \cdot W_c \cdot t_{footing} \cdot l_{heel} + 1.35 EV_{earthBk} - q_{min} \cdot l_{heel} - \frac{1}{2} \cdot \left(q_{heelLC4StrI} - q_{min}\right) \cdot l_{heel}$$

$$V_{uHeelLC4StrI} = 22.6 \cdot \frac{kip}{ft}$$

Moment demand at the critical section

$$M_{HeelDemand} := \max\left(M_{rLC1StrI}, M_{rLC3StrI\_noLL}, M_{rLC3StrI}, M_{rLC4StrI\_noLS}, M_{rLC4StrI}\right) = 127.15 \cdot \frac{kip \cdot ft}{ft}$$

Shear demand at the critical section

$$V_{HeelDemand} := \max\left(V_{uHeelLC1StrI}, V_{uHeelLC3StrI\_noLL}, V_{uHeelLC3StrI}, V_{uHeelLC4StrI\_noLS}, V_{uHeelLC4StrI}\right)$$

$$V_{HeelDemand} = 22.9 \cdot \frac{kip}{ft}$$

Flexural Resistance

LRFD 5.6.3.2

The design procedure consists of calculating the reinforcing steel area required to satisfy the moment demand and checking the selected steel area against the requirements and limitations for developing an adequate moment capacity, controlling crack width, and managing shrinkage and temperature stresses.

As a trial, select No. 9 bars. $bar := 9$

Nominal diameter of a reinforcing steel bar $d_{bar} := Dia(bar) = 1.13 \cdot in$

Cross-section area of a bar on the flexural tension side $A_{bar} := Area(bar) = 1 \cdot in^2$

The spacing shall not exceed 12 in. when the footing thickness is greater than 18 in.

LRFD 5.10.6

Footing thickness $t_{footing} = 3\,ft$

Selected spacing of reinforcing steel bars $s_{bar} := 10 \cdot in$

Area of tension steel provided in a 1-ft wide strip $A_{sProvided} := \frac{A_{bar} \cdot 12in}{s_{bar}} = 1.2 \cdot in^2$

Effective depth $d_e := t_{footing} - Cover_{ft} = 32 \cdot in$

Resistance factor for flexure $\phi_f := 0.9$ **LRFD 5.5.4.2**

A 1-ft wide strip is selected for the design.

Width of the compression face of the section $b := 12in$

Stress block factor $\beta_1 = 0.85$

Solve the following equation of A_s to calculate the required area of steel to satisfy the moment demand. Use an assumed initial A_s value to solve the equation.

Initial assumption $A_s := 1in^2$

Given $M_{HeelDemand} \cdot ft = \phi_f \cdot A_s \cdot f_y \cdot \left[d_e - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b}\right)\right]$

Required area of steel $A_{sRequired} := \text{Find}(A_s) = 0.91 \cdot in^2$

Check if $A_{sProvided} > A_{sRequired}$ $Check := \text{if}\left(A_{sProvided} > A_{sRequired}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Moment capacity of the section with the provided steel $M_{Provided} := \phi_f \cdot A_{sProvided} \cdot f_y \cdot \frac{\left[d_e - \frac{1}{2} \cdot \left(\frac{A_{sProvided} \cdot f_y}{0.85 \cdot f_c \cdot b}\right)\right]}{ft}$

$M_{Provided} = 166.45 \cdot \frac{kip \cdot ft}{ft}$

Distance from the extreme compression fiber to the neutral axis $c := \frac{A_{sProvided} \cdot f_y}{0.85 \cdot f_c \cdot \beta_1 \cdot b} = 2.77 \cdot in$

Check the validity of the assumption, $f_s = f_y$ $Check\_f_s := \text{if}\left(\frac{c}{d_e} < 0.6, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Limits for Reinforcement

LRFD 5.6.3.3

The tensile reinforcement provided must be adequate to develop a factored flexural resistance at least equal to the lesser of the cracking moment or 1.33 times the factored moment from the applicable strength limit state load combinations.

Flexural cracking variability factor $\gamma_1 := 1.6$ For concrete structures that are not precast segmental

Ratio of specified minimum yield strength to ultimate tensile strength of the nonprestressed reinforcement $\gamma_3 := 0.67$ For ASTM615 grade 60 reinforcement

Section modulus $S_c := \frac{1}{6} \cdot b \cdot t_{footing}^2 = 2.59 \times 10^3 \cdot in^3$

Cracking moment $M_{cr} := \dfrac{\gamma_3 \cdot \gamma_1 \cdot f_r \cdot S_c}{ft} = 96.25 \cdot \dfrac{kip \cdot ft}{ft}$

1.33 times the factored moment demand $1.33 \cdot M_{HeelDemand} = 169.12 \cdot \dfrac{kip \cdot ft}{ft}$

The factored moment to satisfy the minimum reinforcement requirement $M_{req} := \min\left(1.33 M_{HeelDemand}, M_{cr}\right) = 96.25 \cdot \dfrac{kip \cdot ft}{ft}$

Check the adequacy of section capacity $Check := if\left(M_{Provided} > M_{req}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Control of Cracking by Distribution of Reinforcement

LRFD 5.6.7

Limiting the width of expected cracks under service conditions extends the service life. The width of potential cracks can be minimized through proper placement of the reinforcement. The check for crack control requires that the actual stress in the reinforcement should not exceed the service limit state stress.

The spacing requirement for the mild steel reinforcement in the layer closest to the tension face $s \le \dfrac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$ **LRFD Eq. 5.6.7-1**

Exposure factor for the Class 1 exposure condition $\gamma_e := 1.00$

Distance from extreme tension fiber to the center of the closest flexural reinforcement $d_c := Cover_{ft} = 4 \cdot in$

Ratio of flexural strain at the extreme tension face to the strain at the centroid of the reinforcement layer closest to the tension face $\beta_s := 1 + \dfrac{d_c}{0.7\left(t_{footing} - d_c\right)} = 1.18$

The calculation of tensile stress in nonprestressed reinforcement at the service limit state, f_{ss}, requires establishing the neutral axis location and the moment demand at the critical section.

The position of the cross-section's neutral axis is determined through an iterative process to calculate the actual stress in the reinforcement. This process starts with an assumed position of the neutral axis as shown below.

Assumed distance from the extreme compression fiber to the neutral axis $x := 5 \cdot in$

Given $\dfrac{1}{2} \cdot b \cdot x^2 = \dfrac{E_s}{E_c} \cdot A_{sProvided} \cdot \left(d_e - x\right)$

Position of the neutral axis $x_{na} := Find(x) = 6.4 \cdot in$

Maximum and minimum bearing pressure under Service I limit state (from the toe design) $q_{maxSerI} = 4.69 \cdot ksf$ $\quad q_{minSerI} = 1.82 \cdot ksf$

Bearing pressure at the critical section $q_{HeelSerI} := q_{minSerI} + \dfrac{\left(q_{maxSerI} - q_{minSerI}\right)}{B_{footing}} \cdot l_{heel} = 3.38 \cdot ksf$

The moment at the critical section under Service I limit state

$$M_{heelSerI} := W_c \cdot t_{footing} \cdot \frac{l_{heel}^2}{2} + EV_{earthBk} \cdot \frac{l_{heel}}{2} \ldots$$
$$+ V_{LSFooting} \cdot \frac{l_{heel}}{2} - q_{minSerI} \cdot \frac{l_{heel}^2}{2} - \left(q_{HeelSerI} - q_{minSerI}\right) \cdot \frac{l_{heel}^2}{6}$$

$$M_{heelSerI} = 41.33 \cdot \frac{kip \cdot ft}{ft}$$

Tensile force in the reinforcing steel due to the service limit state moment

$$T_s := \frac{M_{heelSerI}}{d_e - \frac{x_{na}}{3}} \cdot ft = 16.6 \cdot kip$$

Stress in the reinforcing steel due to the service limit state moment

$$f_{ss1} := \frac{T_s}{A_{sProvided}} = 13.84 \cdot ksi$$

f_{ss} (not to exceed $0.6f_y$)

$$f_{ss} := \min\left(f_{ss1}, 0.6f_y\right) = 13.84 \cdot ksi$$

Required reinforcement spacing

$$s_{barRequired} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{ss}} - 2 \cdot d_c = 34.92 \cdot in$$

Check if the spacing provided < the required spacing

$$Check := if\left(s_{bar} < s_{barRequired}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Shrinkage and Temperature Reinforcement Requirement

LRFD 5.10.6

The required minimum shrinkage and temperature reinforcement area was calculated previously for the toe.

Required shrinkage and temperature steel area

$$A_{shrink.temp} = 0.33 \cdot in^2$$

Check if the provided area of steel > the required area of shrinkage and temperature steel

$$Check := if\left(A_{sProvided} > A_{shrink.temp}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Design for Shear

The critical section for shear in the heel is located at the back face of the abutment wall.

LRFD C5.12.8.6.1

Shear demand at the critical section (max. from the load cases)

$$V_{HeelDemand} = 22.9 \cdot \frac{kip}{ft}$$

Effective width of the section

$$b = 12 \cdot in$$

Depth of the equivalent rectangular stress block

$$a := \frac{A_{sProvided} \cdot f_y}{0.85 \cdot f_c \cdot b} = 2.35 \cdot in$$

Effective shear depth $d_v := \max\left(d_e - \frac{a}{2}, 0.9 \cdot d_e, 0.72 \cdot t_{footing}\right) = 30.82 \cdot in$ **LRFD 5.7.2.8**

The simplified procedure for nonprestressed sections can be used for the design of shear in concrete footings when the distance from the point of zero shear to the face of the wall is less than $3d_v$. **LRFD 5.7.3.4.1**

Check if the distance l_{heel} is less than $3d_v$ $Check := if\left(l_{heel} < 3 \cdot d_v, "Yes", "No"\right) = "No"$

Therefore, the simplified procedure is used.

Factor indicating the ability of diagonally cracked concrete to transmit tension and shear $\beta := 2$

Nominal shear resistance of concrete, V_n, is calculated as follows:

$V_{c1} := 0.0316 \cdot \beta \cdot \sqrt{f_c \cdot ksi} \cdot b \cdot d_e = 42 \cdot kip$ **LRFD Eq. 5.7.3.3-3**

$V_{c2} := 0.25 f_c \cdot b \cdot d_e = 288 \cdot kip$ **LRFD Eq. 5.7.3.3-2**

$V_n := \min\left(V_{c1}, V_{c2}\right) = 42.03 \cdot kip$

Resistance factor for shear $\phi_v := 0.9$ **LRFD 5.5.4.2**

Factored shear resistance (capacity) $V_r := \phi_v \cdot V_n = 37.83 \cdot kip$

Check if the shear capacity is greater than the demand $Check := if\left(\frac{V_r}{ft} > V_{HeelDemand}, "OK", "Not OK"\right) = "OK"$

Development Length of Reinforcement

The flexural reinforcing steel must be developed on each side of the critical section for its full development length. **LRFD 5.10.8.1.2**

Available length for rebar development $l_{d.available} := l_{heel} - Cover_{ft} = 8.92\ ft$

Basic development length $l_{db} := 2.4 \cdot d_{bar} \cdot \frac{f_y}{\sqrt{f_c \cdot ksi}} = 7.82\ ft$ **LRFD Eq. 5.10.8.2.1a-2**

Reinforcement location factor $\lambda_{rl} := 1.3$ More than 12 in. concrete below

Coating factor $\lambda_{cf} := 1.5$

Reinforcement confinement factor $\lambda_{rc} := 0.4$

Excess reinforcement factor $\lambda_{er} := \frac{A_{sRequired}}{A_{sProvided}} = 0.76$ **LRFD Eq. 5.10.8.2.1c-4**

Factor for normal weight concrete $\lambda := 1$

Required development length $l_{d.required} := l_{db} \cdot \frac{\left(\lambda_{rl} \cdot \lambda_{cf} \cdot \lambda_{rc} \cdot \lambda_{er}\right)}{\lambda} = 4.61\ ft$ **LRFD Eq. 5.10.8.2.1a-1**

Check if $l_{d.available} > l_{d.required}$ $Check := if\left(l_{d.available} > l_{d.required}, "OK", "Not OK"\right) = "OK"$

Shrinkage and Temperature Reinforcement Design

The following calculations check the required amount of reinforcing steel in the secondary direction to control shrinkage and temperature stresses in the footing.

The reinforcement along the longitudinal direction of the footing at the top and bottom should satisfy the shrinkage and temperature reinforcement requirements. **LRFD 5.10.6**

The spacing of reinforcement shall not exceed 12 in. when the footing thickness is greater than 18 in. **LRFD 5.10.6**

Note: MDOT practice is to use 18 in. as the maximum spacing. **BDG 5.16.01**

As a trial, select No. 6 bars. $bar := 6$

Nominal diameter of a reinforcing steel bar $d_{bST} := Dia(bar) = 0.75 \cdot in$

Cross-section area of the bar $A_{barST} := Area(bar) = 0.44 \cdot in^2$

Selected bar spacing $s_{barST} := 12 \cdot in$

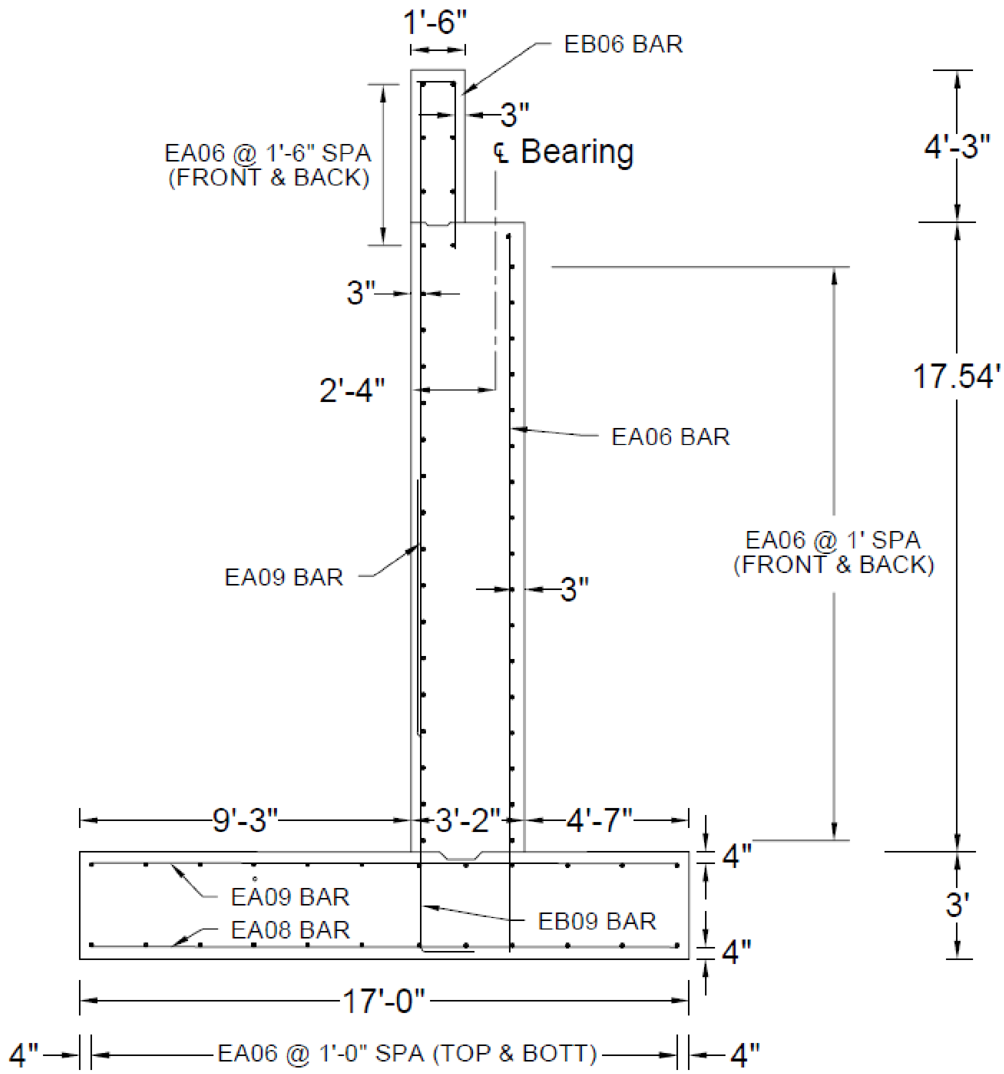
Reinforcing steel area provided in the section $A_{sProvidedST} := \dfrac{A_{barST} \cdot 12in}{s_{barST}} = 0.44 \cdot in^2$

Required minimum area of shrinkage and temperature reinforcement in the footing $A_{shrink.temp} = 0.33 \cdot in^2$

Check if the provided steel area > the required area for shrinkage and temperature steel $Check := if\left(A_{sProvidedST} > A_{shrink.temp}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

The footing design presented in this step provides the following details:

- No. 9 bars at 10.0 in. spacing ($A_s = 1.0$ in.^{2}/ft) as the transverse flexural reinforcement at the top of the footing.
- No. 8 bars at 12.0 in. spacing ($A_s = 0.79$ in.^{2}/ft) as the transverse flexural reinforcement at the bottom of the footing.
- No. 6 bars at 12.0 in. spacing ($A_s = 0.44$ in.^{2}/ft) as the longitudinal shrinkage and temperature reinforcement at the top and bottom of the footing.



Note: Refer to MDOT Bridge Design Guides for additional bars, laps, embedment, and keyway dimensions. They are not shown in this drawing for clarity of main reinforcement.

Appendix 2.A Braking Force and Wind Load Calculation

Description

This appendix presents the braking force and wind load calculation procedures for illustrative purposes.

Braking Force

Since the abutments have expansion bearings, the braking force along the longitudinal direction of the bridge is resisted by the fixed bearings at the pier.

The braking force (BR) shall be taken as the greater of: **LRFD 3.6.4**

- 25% of the axle weight of the design truck / tandem
- 5% of the design truck / tandem weight plus lane load

The braking force is applied on all design lanes assuming that the bridge carries traffic in one direction.

Braking force per lane due to 25% of the axle weight of the design truck / tandem

$$BR_1 := 25\% \cdot (32kip + 32kip + 8kip) = 18 \cdot kip$$

Braking force per lane due to 5% of the design truck / tandem weight plus plus lane load

$$BR_2 := 5\% \cdot \left(72kip + 0.64 \frac{kip}{ft} \cdot 2L_{span} \right) = 10 \cdot kip$$

Note: The MDOT practice, as reflected in the BDS, is to take only 5% of the design truck plus lane load as the breaking force. In addition, the HL-93 modification factor is not included in the braking force calculation. This example describes the MDOT practice.

Braking force selected for the design $BRK := BR_2 = 10 \cdot kip$

The braking force transmitted to the bearings based on the number of lanes with the live load.

Braking force due to 1 loaded lane $BRK_{1L} := BRK \cdot MPF(1) = 12 \cdot kip$

Braking force due to 2 loaded lanes $BRK_{2L} := 2BRK \cdot MPF(2) = 20 \cdot kip$

Braking force due to 3 loaded lanes $BRK_{3L} := 3BRK \cdot MPF(3) = 25.5 \cdot kip$

Braking force due to 4 loaded lanes $BRK_{4L} := 4BRK \cdot MPF(4) = 26 \cdot kip$

Braking force due to 5 loaded lanes $BRK_{5L} := 5BRK \cdot MPF(5) = 32.5 \cdot kip$

The braking force is assumed to be equally shared by the bearings at the pier.

Wind Load

Since the expansion bearings are located over the abutments, the longitudinal component of the superstructure wind load is resisted by the fixed bearings at the bent.

Wind Load on Superstructure

LRFD 3.8.1.1, 3.8.1.2

To calculate the wind load acting on the superstructure, the total depth from the top of the barrier to the bottom of the girder is required. Once the total depth is known, the wind exposure area is calculated. The wind pressure and the exposure area are used to calculate the wind load.

Total depth of the superstructure $D_{total} := h_{Railing} + t_{Deck} + t_{Haunch} + d_{Girder} = 7.08\ ft$

Span length for the superstructure wind load on the abutment $L_{Wind} := \frac{L_{span}}{2} = 50\ ft$

Effective wind area for the superstructure wind load on the abutment $A_{WindSuper} := D_{total} \cdot L_{Wind} = 354.17\ ft^2$

Basic wind speed (mph) $V_w := 115$ **LRFD 3.8.1.1**

Gust effect factor $Gust := 1$ **LRFD Table 3.8.1.2.1-1, no sound barrier**

Drag coefficient, superstructure $C_{DSup} := 1.1$ **LRFD Table 3.8.1.2.1-2**

Superstructure height (ft), assuming that the structure height is less than 33 ft $Z := 33$

Wind exposure category B

Pressure exposure and elevation coefficient for Strength III and Service IV load combinations

$$K_{ZSup} := \frac{\left(2.5 \cdot \ln\left(\frac{Z}{0.9832}\right) + 6.87\right)^2}{345.6} = 0.71$$

LRFD Eq. 3.8.1.2.1-2

Wind pressure on superstructure, Strength III, Service IV (ksf)

$$P_{ZSup.StrIII.ServIV} := 2.56 \cdot 10^{-6} \cdot K_{ZSup} \cdot V_w^{\ 2} \cdot Gust \cdot C_{DSup} = 0.03$$

Wind pressure on superstructure, Strength V, Service I (ksf)

$$P_{ZSup.StrV.ServI} := 2.56 \cdot 10^{-6} \cdot V_w^{\ 2} \cdot Gust \cdot C_{DSup} = 0.04$$

LRFD Eq. 3.8.1.2.1-1

The wind load from the superstructure transmitted to the abutment depends on the attack angle of the wind. The attack angle is measured from a line perpendicular to the girder longitudinal axis. **LRFD 3.8.1.2.2**

Since the span length and height of this girder bridge are less than 150 ft and 33 ft respectively, the following wind load components are used: **LRFD 3.8.1.2.3a**

- Transverse: 100 percent of the wind load calculated based on wind direction perpendicular to the longitudinal axis of the bridge.
- Longitudinal: 25 percent of the transverse load.

The transverse component of the wind load acting on the abutment

$$WS_{STran.StrIII.ServIV} := P_{ZSup.StrIII.ServIV} \cdot ksf \cdot A_{WindSuper} = 9.35 \cdot kip$$

$$WS_{STran.StrV.ServI} := P_{ZSup.StrV.ServI} \cdot ksf \cdot A_{WindSuper} = 13.19 \cdot kip$$

Wind Load on Substructure

The wind pressure on the abutment wall is ignored since the wall is usually shielded from wind by wingwalls or an embankment fill.

Wind Load on Live Load

Since the span length and height of this girder bridge are less than 150 ft and 33 ft respectively, the following wind load components are used: **LRFD 3.8.1.3**

- 0.10 klf, transverse
- 0.04 klf, longitudinal

The transverse and longitudinal components of the wind load acting on the live load and transmitted to the abutment

$$WL_{Tran} := 0.1 \frac{kip}{ft} \cdot L_{Wind} = 5 \cdot kip$$

$$WL_{Long} := 0.04 \frac{kip}{ft} \cdot L_{Wind} = 2 \cdot kip$$

Appendix 2.B Sliding Resistance Check for Spread Footings on Clay

Description

This appendix presents the calculation procedure for checking the sliding resistance of spread footings located on a clay layer.

Undrained shear strength (provided by the Geotechnical Service Section) $S_u := 1.5ksf$

For footings that rest on clay, where footings are supported on at least 6.0 in. of compacted granular material, the sliding resistance may be taken as the lesser of

LRFD 10.6.3.4

- the cohesion of the clay, or
- one-half the normal stress on the interface between the footing and soil.

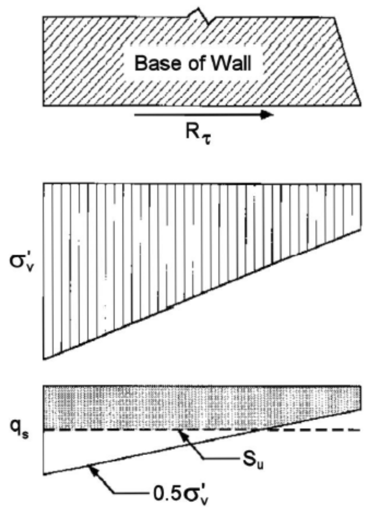


Figure 10.6.3.4-1—Procedure for Estimating Nominal Sliding Resistance for Walls on Clay

The strength limit states are used for this check. Since the resistance is proportional to the vertical loads, the following conditions are used.

- Live load on the bridge is excluded.
- Minimum load factors are used for all vertical loads.
- Maximum load factors are used for the loads that contribute to the horizontal sliding forces.
- Since DW is the future wearing surface load, it is excluded from all load combinations.

Resistance factor for sliding $\phi_\tau := 0.85$ **LRFD Table 10.5.5.5.2-1**

According to the loads in the summary tables provided at the end of Step 2.5, LC I or IV could control the design. Therefore, both load cases are checked.

Load Case I

Factored shear force parallel to the transverse axis of the footing $V_{uFtLC1StrI} = 16.59 \cdot \frac{kip}{ft}$

Factored sliding force (demand) $V_{sliding} := V_{uFtLC1StrI} = 16.59 \cdot \frac{kip}{ft}$

Minimum vertical load $F_{VFtLC1StrIMin} = 41.63 \cdot \frac{kip}{ft}$ **From Section 2.6, sliding resistance check**

Eccentricity in the footing width direction $e_B := \frac{M_{uFtLC1StrI}}{F_{VFtLC1StrIMin}} = 2.11 \cdot ft$

Maximum and minimum bearing pressure

$$q_{max} := \frac{F_{VFtLC1StrIMin}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}}\right) = 4.27 \cdot ksf$$

$$q_{min} := \frac{F_{VFtLC1StrIMin}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}}\right) = 0.62 \cdot ksf$$

Width of the footing with a normal stress greater than $2S_u$

$$B_{Su} := B_{footing} \cdot \frac{q_{max} - 2 \cdot S_u}{q_{max} - q_{min}} = 5.93\ ft$$

Sliding resistance (capacity)

$$V_{resistance} := \phi_T \cdot \left[B_{Su} \cdot S_u + \frac{1}{2} \cdot \left(B_{footing} - B_{Su}\right)\left(\frac{1}{2} q_{min} + S_u\right)\right]$$

$$V_{resistance} = 16.09 \cdot \frac{kip}{ft}$$

Check if $V_{resistance} > V_{sliding}$

$$Check := if\left(V_{resistance} > V_{sliding}, \text{"OK"}, \text{"Not OK"}\right) = \text{"Not OK"}$$

The sliding resistance is inadequate. Since MDOT typically does not use keyways, consider widening the footing to enhance the sliding resistance. When the footing width is too excessive and uneconomical, consider using EPS as a backfill material.

Load Case IV

Factored shear force parallel to the transverse axis of the footing

$$V_{uFtLC4StrI} = 19.85 \cdot \frac{kip}{ft}$$

Factored sliding force (demand)

$$V_{sliding} := V_{uFtLC4StrI} - 1.75 P_{LSFooting} = 16.73 \cdot \frac{kip}{ft}$$

Minimum vertical load

$$F_{VFtLC4StrIMin\_noLS} = 46.72 \cdot \frac{kip}{ft}$$

From Section 2.6, sliding resistance check

Eccentricity in the footing width direction

$$e_B := \frac{M_{uFtLC4StrI\_noLS}}{F_{VFtLC4StrIMin\_noLS}} = 2.5 \cdot ft$$

Maximum and minimum bearing pressure

$$q_{max} := \frac{F_{VFtLC4StrIMin\_noLS}}{B_{footing}} \cdot \left(1 + \frac{6 \cdot e_B}{B_{footing}}\right) = 5.17 \cdot ksf$$

$$q_{min} := \frac{F_{VFtLC4StrIMin\_noLS}}{B_{footing}} \cdot \left(1 - \frac{6 \cdot e_B}{B_{footing}}\right) = 0.33 \cdot ksf$$

Width of the footing with a normal stress greater than $2S_u$

$$B_{Su} := B_{footing} \cdot \frac{q_{max} - 2 \cdot S_u}{q_{max} - q_{min}} = 7.62\ ft$$

Sliding resistance (capacity)

$$V_{resistance} := \phi_T \cdot \left[B_{Su} \cdot S_u + \frac{1}{2} \cdot \left(B_{footing} - B_{Su}\right)\left(\frac{1}{2} q_{min} + S_u\right)\right]$$

$$V_{resistance} = 16.34 \cdot \frac{kip}{ft}$$

Check if $V_{resistance} > V_{sliding}$

$$Check := if\left(V_{resistance} > V_{sliding}, \text{"OK"}, \text{"Not OK"}\right) = \text{"Not OK"}$$

The sliding resistance is inadequate. Since MDOT typically does not use keyways, consider widening the footing to enhance the sliding resistance. When the footing width is too excessive and uneconomical, consider using EPS as a backfill material.